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Shaking table test techniques and fault rupture box testing for SSI
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SSI = Soil Structure Interaction
FHT = Fast Hybrid Testing
FSCS = Full State Control via Simulation
RTDS = Real Time Dynamic Sub-structuring
LMS = Least Mean Squares
FIR = Finite Impulse Response (a finite impulse response (FIR) filter is a filter whose
      impulse response, or response to any finite length input, is of finite duration)
1 INTRODUCTION

1.1 TASK AIMS

1.1.1 Implementation of FHT technique to SSI problems

Due to the limited dimension of the shaking tables, full-scale dynamic tests can be performed in a very limited number of cases: high voltage equipment such as circuit breakers, disconnectors, voltage and current transformers, small power transformers, as civil structures are too massive to be tested in full-size. For this reason, specimens of reduced scale compared to the prototype are typically used in shaking table experiments. To overcome this problem, the Fast Hybrid Testing (FHT) technique can be successfully applied by mathematically modelling the soil, the foundation and the big structure and by testing on the shaking table small parts of the main structure as joints, bearings, beam to column connections, column footings. Moreover the FHT technique could be successfully adopted to validate the mathematical models of reduced-scale models by comparing the mathematical results with the experimental results obtained by testing on the shaking table. In any case the limitations of the reduced-scale physical modelling should be taken into account when dealing with structures and materials, first of all soils, having a non-linear behaviour.

The main aim of task JRA 3.4 was the implementation of the FHT technique to full-size Soil-Structure Interaction (SSI) tests that are performed on the shaking table. This technology has never been implemented up to now for shaking table-based SSI applications. The task focused on extending the FHT technique so it can be implemented at existing shaking tables in Europe. The test beds at UNIVBRIS and NTUA shaking tables were used for this purpose.

The FHT technique can be applied in two variants for SSI problems:
First Variant
The foundation and the soil (up to semi-infinite extend) are simulated as a numerical substructure. On the contrary, the superstructure (e.g. building, bridge pier etc) is described by a physical model on the shaking table. This variant was implemented at NTUA and UNIVBRIS.

Second Variant
The foundation and the soil are modelled using a laminar shear box on the shaking table. A numerical model is used for the superstructure. A laminar shear-box on the shaking table is used for the response of the soil. The feasibility of the application of this variant for SSI problems was investigated at UNIVBRIS and it was concluded that its implementation is extremely difficult and cannot be achieved within the framework of JRA3.4. For this reason, it was decided during the JRA3 Meeting on September 25, 2012 in Lisbon to leave this issue as a future challenge and to concentrate on the first variant of FHT for the needs of JRA3.

The developed FHT techniques at NTUA and UNIVBRIS were applied to a linear SSI problem involving a SDOF superstructure and one – DOF soil (horizontal direction). Different physical models were used in the two Laboratories, in particular, a “heavy” specimen was used at NTUA and a “light” one at UNIVBRIS. However, both specimens had similar dynamic characteristics, as decided during the 1st JRA3 Meeting of October 16, 2009 in Athens, specifically:

- Foundation mass, \( m_0 \), to superstructure mass, \( m_1 \), ratio: \( m_0/m_1 = \frac{1}{4} \).
- Aspect ratio: \( h/B = 0.5, 1.0, 5.0 \), where \( h \) is the height at which the centre of mass of the superstructure is located and \( B \) is the width of the foundation.
- Main period of the fixed-base superstructure: \( T_{str} = 0.2 \) sec, 0.5 sec.
- Main period of soil column: \( T_{soil} > 0.7 \) sec
- Input motion: a) sinusoidal and b) real record.

The application of the FHT technique for SSI problems at NTUA is presented in Section 3 and at UNIVBRIS in Section 4.

1.1.2 Soil containers for strong motion testing

The physical models for the investigation of the dynamic response of a soil structure interaction (SSI) system makes use of a soil container designed to work in conjunction with a shaking table.
Over the past forty years, the use of such containers has become commonplace. Soil containers generally consist of hollow boxes secured at their base to a shaking table and filled with a test soil onto or into which the structure is placed or embedded. To avoid boundary effects, flexible ‘shear stacks’ that allow the ends of the container to flex are used.

The use of laminar boxes (for a definition see section 2.1.1) in strong motion testing, in which large shear deformations are induced by the strong seismic shaking, and the effect of boundary reflections were investigated at UNIVBRIS and IZIIS. The scope was to determine the influence of the stiffness and inertia of the laminates and to devise a simplified methodology for numerical simulation. Shaking table results were compared with centrifuge test results, as well as with numerical results, in order to determine the influence of the side reflections on the soil impedances (especially the compression wave impedance of the soil). Additionally, the comparisons allowed devising a simplified methodology to isolate the effect of boundary reflections and exploring the potential of actively controlled boundary schemes.

This work is still on-going. The up-to-day investigation is presented in Section 2.

1.1.3 Investigation of fault rupture propagation

Fault-rupture propagation and its interaction with the foundation – structure systems were investigated using the existing fault rupture box (for a definition see section 5.2.1) at NTUA. The results were used to develop a simplified methodology to compute the faulting-induced stressing on foundations and structures.

1.2 FAST HYBRID TESTING (FHT) TECHNIQUE

Substructure testing divides the system under investigation into two linked subsystems. The part of experimental interest is tested physically while the remaining easily modelled part is simulated numerically. The physical and numerical substructures exchange information in real time allowing the test to proceed as if the system was a whole. This methodology allows the dynamic behaviour of large or even full scale systems to be investigated. Until now the dynamic behaviour of anti-vibration technologies (such as isolators, bearings and dampers) and structural components (such as beams, columns and joints) have been investigated by this method.
Actuation in such tests is usually via servo-hydraulic actuators that act directly on the physical part. However, in civil engineering, there can be a requirement to model systems with distributed mass. Then, the physical substructure must be tested on a shaking table in order to correctly capture the distribution of inertia. Herein, the substructure test method as applied to a shaking table actuation is considered.

1.2.1 Real–time dynamic substructuring

In structural testing and experimental earthquake engineering a lot of attention has been recently focused on hybrid testing (HT) methods (Saouma and Sivaselvan 2008). HT generally refers to the process of creating a real–time hybrid model of a structure, by combining (at least) one numerical model (referred to as the numerical substructure) with an experimental specimen (referred to as the physical substructure). Rather than testing a single specimen, which may be part of an entire structure, using well–known and established techniques (i.e. a shaking table), HT incorporates a surrounding environment that attempts to reproduce the effects of the total structure on that single specimen. This scheme provides a better understanding about the underlying dynamics, as it realizes a more realistic testing framework.

In essence, HT can be considered as a special case of dynamic substructuring (DS), a framework that has been developed over the last 40 years and allows the detailed evaluation of complex structural systems by analyzing the performance of local subsystems (De Klerk et al. 2008). The involved HT techniques can be either based on a concept similar to the pseudodynamic testing, or can resemble the hardware–in–the–loop scheme, utilized for the testing of control systems and mechanical structures (Shing 2008). In this latter case, HT is also referred to as Real–Time Dynamic Substructuring (RTDS).

Among other critical issues needed to be determined (such as the numerical integration scheme), RTDS tests require the realization of a so called transfer system. The latter is a set of controllers, actuators, sensors and A/D – D/A devices, which permit the communication between the numerical and the physical substructure. This interaction involves the successful implementation of the following steps, which are implemented by the majority of the current RTDS methods (Wagg et al. 2008):
1. The displacements at the interface are calculated by the numerical substructure(s).
2. These displacements are transferred to the physical substructure by means of the transfer system.
3. The resulting reaction forces of the physical substructure are measured.
4. These forces are fed back to the numerical model and, together with the excitation, are used to the calculation of displacements at the interface for the next time interval.

Since the process is taking place in real – time, these steps must be completed within one sample period. Under the presence of the transfer system, this hard constraint may pose significant difficulties. More specifically, the transfer system is dominated by its own dynamics, which interfere between the numerical and the physical substructure. These dynamics, which naturally include a transfer delay, must be compensated in order to retain consistency with the steps indicated above (Wallace 2006). The problem is much more evident when a shaking table is selected to be the transfer system, as in this case the transfer dynamics are much more complicated. In addition, focusing on the SSI problem, the application of RTDS tests remains an active challenge (Wang et al. 2011).

An issue of concern is the measurement of the reaction forces (step 3). Typically, load cell transducers are used as the interface between the numerical and physical parts, measuring the reaction forces resulting from physical part which are fed back to the numerical part. This approach was followed at the implementation of FHT at UNIVBRIS. However, due to the large weight of the specimen that was used at NTUA, load cells caused operational problems, since they could not supply full fixation of the specimen to the shaking table. For this reason, at NTUA the force feedback was measured using an accelerometer that was placed on the specimen mass.

In general, multi axis force plates that are capable of measuring the dynamic feedback force are expensive and influence the design of the physical substructure. Acceleration feedback provides a convenient alternative since it uses standard equipment commonly available in earthquake engineering research infrastructures. However, acceleration feedback can be used only for those physical substructures which can be represented as lumped mass systems. Distributed mass structures are commonplace in civil engineering. When testing such distributed mass structures, a force plate is essential.
1.2.2 Model of the shaking table system

Transfer systems are used to impose force or displacement on a test specimen and to imitate the constraint dynamics between the numerical model and physical model. To achieve near-perfect synchronization of the interface response between the physical specimen and the numerical model, a good controller is needed to compensate for the dynamics of the transfer systems. Transfer systems are typically single actuators (electric or hydraulic) and include their proprietary controller such as PID controller. With the exception of a phase lag, an actuator normally has a good performance at low frequency range (e.g., 0-10 Hz). To improve the phase response a number of delay compensation algorithms have been proposed. Compared with a single actuator a shaking table has more complex dynamic characteristics. The large mass of the shaking table limits the operation frequency bandwidth. Indeed, both a phase lag and a magnitude error exist in the frequency range of interest. Delay compensation cannot adequately compensate for such dynamics. Thus, there is a requirement for the development of a controller for shaking table substructuring that is capable of dealing with these more complex dynamics.

Details on this issue are presented in Section 4.1.

1.2.3 Soil models for application of FHT technique to SSI problems

The response of structures to earthquake motions is strongly dependent upon the performance of the surrounding soil: the response of a rigid soil based structure may differ significantly from the response of a flexible soil based structure. Indeed, soil – structure interaction (SSI) is extremely important, especially when the soil is not assumed to be rigid, as in conventional structural response. As a result of this interaction, the soil becomes subject to deformations, which, in turn, cause a substantial alteration of the structure’s response (Clough and Penzien 2003). Of course, this alteration is insignificant when the soil is very stiff (Datta 2010).

The complete description of the foundation and the surrounding soil is both complicated and computationally expensive. For this type of problems, engineering practice has privileged the development of simplified models for the description of the foundation – soil system behaviour (Chatzigogos et al. 2011), which can be used for the implementation of FHT technique. Focusing on shallow foundations that may be assumed rigid with respect to the underlying soil, there’s a
Shaking table test techniques and fault rupture box testing for SSI

rich literature on simplified models, which may be generally classified under two groups (Figure 1.1): (a) the so-called macroelement models and (b) the models based on the Winkler decoupling hypothesis. In the former case (Paolucci 1997, Cremer et al. 2001), the footing and the soil are considered as a single macroelement and a six degrees – of – freedom (DOF) (three dimensional case) or a three DOF (two dimensional case) model is formulated, describing the resultant force – displacement behaviour of a point (usually the centre) of the footing in the vertical, horizontal and rotational directions. In the latter case (Psycharis and Jennings 1983, Chopra and Yim 1984, Allotey and Naggar 2008, Psycharis 2008), the soil is replaced by a number of decoupled horizontal and vertical springs possessing an appropriate (elastoplastic, contact-breaking etc.) constitutive law. The decoupling hypothesis is convenient because it simplifies considerably the integration of the local spring response to obtain the global footing response. However, it is bound to a number of limitations such as the difficulty to calibrate model parameters and to describe the coupling among the DOF of the footing.

![Diagram](image1)

(a) (b)

**Figure 1.1.** Classes of simplified models for shallow foundations: (a) macroelements, and (b) Winkler – spring models (from Chatzigogos et al. 2011).

Existing macroelement models are based on the assumption that the surface of ultimate loads of the system can be identified as a yield surface of a unique global plasticity model. While this assumption is particularly helpful from the practical point of view, it retains some limitations, since the ultimate surface of the foundation is actually obtained as a combined result of different non-linear mechanisms (soil plasticization, uplift and sliding), which possess their intrinsic characteristics and cannot always be put together into a unique plasticity model framework.
To overcome such limitations, it has been proposed (Chatzigogos et al. 2008, Chatzigogos et al. 2011) to reconstruct the ultimate surface of the foundation using as building blocks the mechanisms that actually produce it, and which explicitly depend on the characteristics of the soil and the soil – footing interface. Each mechanism can be modeled independently from one another and this allows respecting its particular characteristics. Then, the mechanisms that are relevant for each application are put together into the macroelement in a coupled way so that the ultimate surface of the foundation is actually obtained as their combined result.

Figure 1.2 displays a perfectly rigid footing under quasi – static planar loading. The vertical force $N$, the horizontal force $V$ and the moment $M$ act on the footing and produce the corresponding displacements $u_x$ and $u_z$ as well as the rotation $\theta_y$, respectively.

This formulation encompasses a wide variety of combinations of soil and foundation – soil interface conditions that are interesting for practical applications. As noted in Chatzigogos et al. (2011), each non-linear mechanism participating in the global response of the system is modeled independently and the surface of ultimate loads is retrieved as the combined result of all active mechanisms. This allows formulating each mechanism by respecting its particular characteristics and offers the possibility of activating, modifying or deactivating each mechanism according to the context of application. To this, the macroelement model comprises three mechanisms: (a) the mechanism of sliding at the soil – footing interface, (b) the mechanism of soil yielding in the vicinity of the footing and (c) the mechanism of uplift as the footing may get detached from the soil. The first two are irreversible and dissipative and are combined within a multi – mechanism
plasticity formulation. The third mechanism is reversible and non-dissipative, reproduced with a phenomenological nonlinear hyperelastic model. The involved parameters can be easily calibrated and / or replaced by user – specified data. In this framework, the macroelement model can serve as a simplified tool to allow effective SSI. Details are presented in deliverable D14.3.
2 SOIL CONTAINERS FOR STRONG MOTION TESTING

The shortcomings of current structural design methodologies and construction practices are all too often revealed by earthquakes. Post earthquake reconnaissance investigations lead to improvements in engineering analysis, design and construction practices. Earthquake engineers learn from past failures. Prior to assuming that a proposed failure mechanism is valid or applying an untested earthquake resistant design method to a practical problem it is advisable to obtain a verification of the assumed performance. One way that this can be accomplished is through field data. However, due to the infrequency of large earthquakes and a dearth of instrumented structures, this method of verification can prove difficult. Alternatively, a record of observations of the response of a physical system can be acquired by modeling.

2.1 LAMINAR BOXES AT UNIVBRIS

2.1.1 Soil containers for SSI studies

When the dynamic response of a soil structure interaction (SSI) system is of concern, physical modeling makes use of a soil container designed to work in conjunction with a shaking table. Over the past forty years, the use of such containers has become commonplace. Both single-gravity and centrifuge containers have been developed to tackle a wide variety of geodynamic problems: soil-structure interaction, liquefaction, dynamic bearing capacity, slopes and embankments, piles, retaining walls. Soil containers generally consist of hollow boxes secured at their base to a shaking table and filled with a test soil onto or into which the structure is placed or embedded. Rigid boxes have been used and are simple to construct. However, such containers
need to be large to avoid boundary effects upsetting the response of the test structure. To guarantee zero boundary effects, an infinite half space soil box would be needed. An alternative to the rigid box is the flexible ‘shear stack’ that allows the ends of the container to flex. This flexible container allows the use of a smaller soil deposit because it can, to some extent, replicate the free field boundary conditions that would be present were the soil sample under test to be part of a much larger soil deposit – the container becomes invisible to the soil. Moreover, boundary effects must be minimized according to the modeling application. To model liquefaction, for example, the large lateral displacements associated with a liquefied soil must be unimpeded. Thus, the ‘laminar box’ was devised (e.g. Hushmand et al., 1988, Ueng et al., 2006). These containers are constructed from a stack of stiff rings each capable of independent and unrestrained lateral displacement giving the box negligible shear stiffness. Internal walls of the rings are made smooth to restrict boundary shear stresses. Conversely, when liquefaction is not of interest, boundaries must be both frictional to enable complimentary shear stress generation and flexible to allow for natural modulation of the seismic waves.

Zeng & Schofield’s (1996) Equivalent Shear Beam (ESB) centrifuge container, Figure 2.1, was constructed from an alternating stack of aluminium alloy and rubber rings for flexibility. The composite shear stiffness of the ESB was tuned to the dynamic stiffness of a test soil by careful detailing of the rubber layer thickness. This design specification is rather restrictive since soil-stiffness degradation ensures it can be achieved at only a single level of seismic excitation. For other soils and other levels of excitation the ESB malfunctions by either constraining or enhancing the dynamic lateral soil displacements. Crewe et al. (1995) developed a more versatile soil container named a ‘large shear stack’. Their design objective was that the test soil, not the stack, would drive the system response. To this end, the shear stiffness and inertia of the stack were reduced. No experimental verification of Crewe et al.’s (1995) stack has yet been presented. However, a performance evaluation of its smaller precursor is available. The hollow aluminium rings of Dar’s (1993) shear stack are, like the ESB of Figure 2.1, separated by solid rubber rings giving it a fundamental frequency when empty of 12Hz. Dar (1993) used his stack to investigate the degradation of shear stiffness and damping of a dry sand deposit via resonant testing. A limitation of Dar’s (1993) apparatus is that at higher strains the experimentally derived soil stiffness is higher than expected. Crewe et al (1995) reasoned that while the soil is known to
suffer from strain level dependant stiffness degradation, the stack remains elastic and, as a result, increasingly dominates the response.

Figure 2.1. Zeng & Schofield’s (1996) Equivalent Shear Beam soil container.

The latest shear stack incarnation was developed as part of the New Methods of Mitigating the Seismic Risk of Existing Foundations (NEMISREF) project funded by the 5th framework of the European Commission and goes some way toward adapting the design methodology so that functionality can be assured at large shear deformations. The shear stack is a direct descendant of Crewe et al.’s (1995) apparatus. However, it is closer in size to Dar’s (1993) small shear stack: 1.2m long as opposed to 5m, 0.55m wide as opposed to 1m, and 0.8m deep as opposed to 1.2m, significantly reducing the costs and timescales associated with testing. Design details are presented in Figure 2.2.

Figure 2.2. The University of Bristol small shear stack (after Pitilakis et al. 2008).
The apparatus consists of eight aluminium rings, rectangular in plan, which are stacked alternately with rubber sections. The rubber sections span only the end-walls of the stack, not the side-walls, giving a fundamental frequency when empty of 6Hz, significantly less than Dar’s (1993) apparatus. The aluminium rings are constructed from box section to minimise inertia. The stack is secured to the shaking table by its base and shaken horizontally lengthways (in the y direction). Its floor is roughened to aid the transmission of shear waves; the internal end walls are similarly treated to enable complementary shear stresses. The internal side walls are lubricated for plane strain. Not pictured in Figure 2.2 are the rigid steel restraining frame and the system of bearings used to prevent unwanted motion in the x and z-directions.

The idealized test soil response to base acceleration has been presented elsewhere and is as depicted in Figure 2.2(c). Vertically propagating shear waves induce shear stresses within a test soil. Normal stresses are kept constant. Lateral deflection $u$ of the soil column in the y-direction is caused by shear deformation. The shear stress at depth $d$ is the product of the soil density $\rho$ and the integral of lateral acceleration through the overlying soil. Given surface acceleration $\ddot{u}_{d=0}$ and acceleration $\ddot{u}_d$ at depth $d$, first-order finite difference approximations for the shear stress $\tau_{zy}'$ and shear strain $\gamma_{zy}$ at time $t$ are:

$$\tau_{zy}'(d,t) = \rho d(\ddot{u}_d(t) + \ddot{u}_{d=0}(t))/2$$  \hspace{1cm} (2.1)

$$\gamma_{zy}(d,t) = (u_d(t) - u_{d=0}(t))/d$$  \hspace{1cm} (2.2)

The performance of the shear stack was assessed by Pitilakis et al (2008) by adapting the design methodology of Zeng & Schofield’s (1996) ESB soil container. The deformation of a soil element in a soil column subjected to lateral base shaking depends on the stress-strain relationship of the soil. Zeng & Schofield (1996) utilized the Hardin & Drnevich (1972) hyperbolic stress-strain relationship to describe the idealized soil response. This non-linear elastic model relates shear strain $\gamma_{zy}$ to shear stress $\tau_{zy}'$ using limiting values of shear stiffness $G_0$ and shear stress $\tau_{max}'$:

$$\gamma_{zy}(d,t) = \tau_{max}' \tau_{zy}'(d,t) / (G_0 \tau_{max}' - G_0 \tau_{zy}'(d,t))$$  \hspace{1cm} (2.3)
Integration of $\gamma_z$ across the soil depth $d$ gives the lateral displacement $u_d$. $G_0$ can be calculated from Equation 2.3; $\tau'_{zy}$ from Equation 2.1 at defined acceleration magnitudes. An expression for $\tau'_{\text{max}}$ can be derived from a consideration of Mohr’s circles for a soil element of density $\rho$ and strength $\phi'$ undergoing lateral acceleration (Zeng & Schofield, 1996):

$$
\tau'_{\text{max}} (d) = \frac{\rho g d}{2} \sqrt{(\sin \phi' + K' \sin \phi')^2 - (1 - K'_0)^2}
$$

(2.4)

The measured peak lateral deflections of the full ESB container subjected to three magnitudes of earthquake are compared with the idealized response in Figure 2.3(a) (taken from Zeng & Schofield, 1996). At low excitation levels, the stiffness of the soil is higher than that of the ESB and so inertia of the ESB drives the deflections upwards. At high excitations, the stiffness of the soil is lower than that of the ESB and the ESB constrains deflections. At the design earthquake, stiffness and deflection of the soil and ESB match.

![Figure 2.3](image)

**Figure 2.3.** End-wall deflections (symbols) compared to the idealized response calculated at equivalent acceleration levels (dashed lines): (a) Zeng and Schofield’s (1996) ESB; (b) the shear stack.

The performance of the shear stack can be assessed by comparing the deflections of its end wall with the response predicted by the Hardin & Drnevich (1972) model. The assessment is valid at low to medium excitation levels, when induced shear stresses $\tau'_{zy}$ are less than the limiting magnitude given by Equation 2.4. At higher acceleration levels, Equation 2.3 breaks down. For LB 14-25, this occurs at a shaking table acceleration of around 0.38g.
At six magnitudes of sinusoidal excitation, a single cycle of steady-state response was selected from which peak displacements of the shear stack’s end walls were extracted. Peak deflection data are plotted in Figure 2.3(b) alongside the Hardin & Drnevich (1972) predictions. The response is adequately predicted at all excitation magnitudes. Unlike the ESB, it appears that the inertia driven phenomenon of increased deflections when the container to soil stiffness ratio is less than unity is not apparent. This is presumably due to the comparatively small inertia forces induced by the stack’s hollow aluminium rings. In summary, low magnitudes of soil container stiffness and inertia are necessary when the requirement is to test across a wide range of strain. With low stiffness and low mass, the shear stack response is as predicted up until failure.

Shear stack performance at larger deformations was considered by Dietz & Muir Wood (2007) who compared the response of an included soil specimen to three different types of excitation: white noise (indicated with subscript $r$), sinusoidal (indicated with subscript $s$), and pulse (indicated with subscript $p$). The recorded responses were presented on axes of normalized shear modulus ($G/G_0$) and damping (D) against shear strain ($\gamma$) where they were compared to various empirical relationships (Figure 2.4). Unlike Dar’s (1993) stiffer shear stack, the softer stack was able to track the strain level dependent stiffness degradation of the empirical relationships up to large magnitudes of shear strain.

(a)  
(b)

Figure 2.4. The evolution of $G$ and $D$ with shear strain (after Dietz & Muir Wood, 2007).
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It seems that to model the response of soils at both small and large deformations it is necessary to minimize the stiffness and inertia of the soil container. Indeed, it seems as though the requirements for such a soil container are close to those of liquefaction studies where lamellar boxes constructed from a rings guided on bearings that are capable of independent and unrestrained lateral displacement are used (e.g. Hushmand et al., 1988, Ueng et al., 2006). Such containers result in negligible shear stiffness. The following section presents a conceptual design of a new shear stack intended to capitalize on this realization.

2.1.2 The hydraulic shear stack

The main problem with current designs for lamellar boxes is the low manufacturing tolerance that is required for the stack to “shear” without any sticking and the high cost of the bearings. In an ideal case, if large strain behaviour is to be modelled, the rings should be completely free to move during excitation. However during filling the box should remain rigid. Additionally, in for unsymmetrical SSI applications (such as retaining wall testing), it is arguable that the ends of the shear stack should have independently controllable stiffnesses.

Therefore as an alternative to the standard rigid ring model for a shear stack, here a conceptual design of a shear stack is considered that has end walls consisting of a set of horizontal parallel beams each supported by two passive hydraulic pistons. Figure 2.5 shows the hydraulic shear stack concept for a single layer in the stack. A single layer is formed by four linked pistons supporting two beams, one at each end of the stack. With an appropriate arrangement of hydraulic piping and valves, in principle it is possible to lock the stack rigid, to have a fully flexible stack (but with the beams remaining parallel and tracking each other), to have a flexible stack but with increased damping, to have adjustable stiffness in the ends of the stack and finally to have an actuated stack. This system can also be modified to provide the parallel motion of the end beams but with more flexibility over the stiffness of each layer at each end of the stack, as shown in Figure 2.6.
Figure 2.5. Plan view (left) and elevation (right) showing conceptual hydraulic arrangement for a symmetrically filled shear stack.

Figure 2.6. Plan view (left) and elevation (right) showing conceptual hydraulic arrangement for an asymmetrically filled shear stack.
In order to assess the potential for this type of design of a shear stack a simplified Simulink model of a single layer of a hydraulic based shear stack was initially developed (see Figure 2.7). Using this model it was possible to explore the effect of varying the hydraulic pipe diameter and the piston areas.

![Figure 2.7. Initial Simulink model used to study the effect of pipeline flow resistance on two opposing actuators.](image)

In this model the soil was simply modelled as two identical single degree of freedom oscillators, with one piston attached to each mass to link the two systems together through the hydraulic system. The excitation was a sinusoidal force which was applied to each of the masses. If the hydraulic shear stack were working perfectly the motion of the two “soil” masses would be identical and the oil in the hydraulic system would move freely through the system. However the compressibility of the oil, the expansion of the hydraulic pipes when pressurised, friction in the pistons etc. meant that there were losses in the system and the two soil masses did not move as one. In particular this model highlighted a compromise that would have to be made when building such a shear stack. On one hand it is desirable for the pistons and pipework to have a large cross sectional area to minimise friction losses in the pipework, however in order to reduce the flexibility of the system, caused by expansion of the pipe and piston walls under pressure (especially when the system is locked to prevent movement), it is desirable to minimise the cross
sectional areas of the pipework and pistons. These opposing demands on the hydraulic design mean that there will always be some error in motion between the two ends of the stack and the sizing of the hydraulic elements will depend on what is felt to be an acceptable error. In order to better understand this compromise a full model of a single layer of a hydraulic shear stack was developed (as shown in Figure 2.8).

![Simulink model](image)

Figure 2.8. Simulink model used to compare the performance of a single layer ‘ideal’ shear stack with hydraulic end walls.

This model is essentially the same as that shown in Figure 2.7 but with the inclusion of all 4 pistons, with more accurate representation of the actuators (including port sizing), and with a reference SDOF system unconnected to the hydraulics for comparison with the shear stack motions. The performance of this system under a variety of different input was studied and a few key results are presented below.
Figure 2.9 shows the peak acceleration response of the ‘soil’ when it is subjected to random excitation. In this case the SDOF soil model has a natural frequency of 20Hz and a damping of 5% and the ideal and actual responses of the system at various excitation levels are shown. The ideal responses are always higher than the actual responses of the soil in the shear stack because of the inherent damping in the hydraulic system and the error is also dependant on the amplitude of excitation (larger motions result in more movement of the oil and increasing losses due to friction etc.). Figure 2.10 shows the detailed response of the system around 20Hz and it is clear that the ideal response is relatively independent of the excitation amplitude whereas the actual response varies more significantly. In this case the error in response varies from about 5% to about 14% depending on the amplitude of motion.

![Comparison between Ideal and Actual Response of soil deposit for various levels of excitation](image)

**Figure 2.9.** Comparison between the ideal and actual response of the hydraulic shear stack under varying amplitude random excitation.
Whilst a 5% error relative to the perfect case where the soil behaves as if the shear stack does not exist might be considered acceptable it is arguable whether a 14% error is reasonable. Therefore although the concept of a hydraulic shear stack shows some promise, and certainly opens up some experimental options that are not available in current designs, the hydraulics would need very careful design in order to minimise the energy losses in the system and make such a system practicable.
2.2 LAMINAR BOX AT IZIIS

2.2.1 Introduction

IZIIS investigated the key parameters and criteria, which laminar boxes as experimental tools have to satisfy in order to enable representative shaking table tests on large geo-models.

Model tests are essential when the prototype behaviour is complex and difficult to understand. In model testing, usually the boundary conditions of a prototype are reproduced in a small-scale model. If done properly, scaled model tests can be advantageous for seismic studies because of their ability to give economic and realistic information about ground amplification, change in pore water pressure, soil non-linearity, and occurrence of failure and soil structure interaction problems (Prasad et. al, 2004). The model tests can be divided into two categories, namely, those performed under gravitational field of earth (generally called shaking table tests or 1g tests) and those performed under higher gravitational field (centrifuge tests or multi-g tests). Both shaking table and centrifuge model tests have certain advantages and limitations. Shaking table tests have the advantage of well controlled large amplitude, multi-axis input motions and easier experimental measurements and their use is justified if the purpose of the test is to validate the numerical model or to understand the basic failure mechanisms (Jafarzadeh, 2004). In the case of geotechnical structures, an additional issue is related to the presence of a container which will set the boundary conditions of the soil. Laminar box or shear box is widely used experimental tool for both mentioned categories.

The laminar box is designed and is planned to be set in the laboratory for dynamic testing of soils at the Institute for earthquake engineering and engineering seismology IZIIS in Skopje, Macedonia. The laminar box is planned to be used for experimental testing on fully saturated cohesionless soil in order to investigate the liquefaction phenomena and cyclic behavior of cohesionless soil in earthquake conditions.
2.2.2 Design process of the laminar box in IZIIS

The geotechnical model cannot be directly mounted on shaking table due to the requirements of confinement. An ideal container should be large, flexible, massless and transparent. However, it is impossible to provide all the essential features. During the last decade many laminar boxes are being developed across the globe to suit field conditions better. Table 2.1 presents some of the existing laminar boxes around different research centers in the world.

Table 2.1. Some laminar boxes designs (modified after Turan et al.(2009))

<table>
<thead>
<tr>
<th>Author</th>
<th>Dimensions (W x L x H) [mm³]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Gibson</td>
<td>350 x 900 x 470</td>
</tr>
<tr>
<td>Prasad et al.</td>
<td>500 x 1000 x 1000</td>
</tr>
<tr>
<td>Meymand</td>
<td>2280 x 2130 x 2130</td>
</tr>
<tr>
<td>Ueng and Chang</td>
<td>1888 x 1888 x 1520</td>
</tr>
<tr>
<td>Van Laak et al.</td>
<td>254 x 457 x 254</td>
</tr>
<tr>
<td>Pamuk et al.</td>
<td>355 x 710 x 355</td>
</tr>
<tr>
<td>Shen et al.</td>
<td>584 x 500 x 500</td>
</tr>
<tr>
<td>Takanashi et al.</td>
<td>200 x 450 x 325</td>
</tr>
<tr>
<td>A.T. Carvalho et al.</td>
<td>750 x 2000 x 1750</td>
</tr>
<tr>
<td>Jafazaderh F.</td>
<td>1000 x 1000 x 1000</td>
</tr>
<tr>
<td>Wienbroer et al.</td>
<td>600 x 800 x 2100</td>
</tr>
</tbody>
</table>

The geometrical shape of the cross section may be square or circle. Due to symmetry, the circle cross section is more suitable. However, constructing a container having square cross section is easier from the manufacturing point of view. Effects of the corners on the measuring points can be decreased by installing the measuring devices into center cross section of the container.

According to the capacity of the shaking table, the models weighing up to 40 tons can be tested on the shaking table. Thus the square cross section with 2 x 1 m dimensions in plan and 1.5 m in height is defined.

For flexibility in the walls of the container a laminar system is applied, since in this system, the shear stiffness of the walls is limited to the friction between the layers and the influence of
rubber membrane inside the box. So this kind, so called laminar shear box; at the time of liquefaction, has the least undesirable effect in the real behaviour of the model (Sesov, 2003).

The ideal container is one that gives a seismic response of the soil model identical to that obtained in the prototype, i.e. the semi-infinite soil layer 1D response under vertically propagating shear waves. The boundary conditions created by the model container walls have to be considered carefully, otherwise the field conditions cannot be simulated properly.

The presence of rigid and smooth end walls in the case of a ground model introduce three serious boundary effects compared with a semi-infinite soil layer in the prototype: deformation incompatibility, stress dissimilarity and input excitation pattern dissimilarity (A.T. Carvalho et al. 2010).

The present laminar box is designed according the following criteria:

- Layers and the membrane inside should have minimum stiffness to horizontal shear.
- The laminar box should have mass much smaller than the soil material which is built inside
- It retains water and air without leakage.
- It offers little resistance to vertical settlement of soil.
- Height of each layer is small which increased the flexibility for the deformation of soil inside.
- It is fairly large to better simulate field behaviour.
- It possesses capability to increase confining pressure.
- It maintains its horizontal cross section during shaking.
- It develops shear stress on the interface between soil and vertical wall equal to that on the horizontal plane.
- It provides good contact between the bearings and groove.
- It allows free movement of soil along the transverse cross section.
- It possesses provision for instrumentation.
- It is strong and stable against all the dynamic forces and moments.
- To provide stiff connection to the shaking table.
The 3 dimensional layout of the designed container is shown in Figure 2.11. The container consists of the following main components:
(a) Aluminum layers and ball bearings;
(b) Base plate with the saturation and drainage system in the floor;
(c) The upper and the side guides;
(d) Internal membrane used as a cut-off and keeping the moving bearings away from dust.

**Figure 2.11. 3-d view of the laminar box designed in IZIIS.**

### 2.2.2.1 The layers and the mechanism of motion

Each layer is a square ring which is composed of hollow aluminum profiles with 40×80 $mm^2$ section (Fig. 2.12). The whole system is composed of 12 layers-rings with height of 1.5 m. In order to minimize the friction between the layers, transfer ball bearings have been used so that the two dimensional motion in the horizontal plane is possible. These balls are designed in a way that they can be simply as possible and without any additional devices to provide maximum sliding of rings with minimum friction. This makes the balls act as a column between the lower and upper hollow aluminum sections and prevents the surface from being deformed, by the point
contact stress between the ball bearing and the surface of the aluminum profile. In order to make
the distribution uniform, 12 rotating ball bearing are used in each layer and the gap between the 2
adjacent layers is 3 mm.

![Perspective view of the laminar ring.](image)

**Figure 2.12. Perspective view of the laminar ring.**

### 2.2.2.2 Base plate and saturation and drainage systems

The lowest layer has been fixed on a steel base with $2.1 \times 1.5 \times 0.01$ m in dimensions. In order to
allow saturation of the soil model, the base has been designed with double bottom plates (one is
perforated) and system of small pipes (Figure 2.13). The bottom plate of the model is covered
with porous stone. In this way, not only saturation and drainage of the samples is facilitated but
this improves the contact between the soil and the steel plate of the container which makes better
shear stress transition.

For hydraulic cut-off system and the protection of the ball bearings, the inside of the container is
covered by a 2 mm thick rubber membrane.
2.2.2.3 *The upper and side guides*

In the four sides of the box 4 steel columns have been installed, so as to prevent the frames from the oversize deformations while conducting the test. Also, in order to prevent the layers from getting separated at the time of vibration, a horizontal steel cross has been installed above the columns tangent to the ball bearings of the uppermost layer. Horizontal movement of max.±17 cm is allowed.
2.2.3 Construction of the laminar box in IZIIS

All the construction activities are carried out at IZIIS (Figure 2.16 – 2.20) only some necessary specific parts for the project are purchased from specialized companies.

Figure 2.15. Longitudinal section of the laminar box.

Figure 2.16. The construction process of the laminar box.
Shaking table test techniques and fault rupture box testing for SSI

Figure 2.17. View of the constructed laminar box, delivered in IZIIS laboratory.

Figure 2.18. The base saturation plate at the bottom of the laminar box.
Figure 2.19. Cross section of the aluminum profiles & schematic cross section.

Figure 2.20. The sliding ball for frictionless movement of the profiles.
Model testing under 1g environment in earthquake geotechnical engineering has become an integral part of research. The laminar box is a part of investigation study on the liquefaction phenomena and cyclic behaviour of cohesionless soil. Use of laminar box will improve the efficiency of testing and simulating the real ground conditions. Amplification, liquefaction and cyclic mobility phenomenon, excess pore water pressure generation and dissipation rates can be performed using such facilities.

The designed laminar box is going to be used in order to investigate the liquefaction phenomena and cyclic response of cohesionless soils. It represents useful tool for future experimental investigation of different kind of phenomena from the area of earthquake geotechnical engineering. We strongly believe that this new design of Laminar Container will overcome a lot of shortcomings that previous types of laminar box or shear box exhibits. The results from the planned investigations will have big influence in the European region for the development of the geotechnical earthquake engineering.

2.3 INVESTIGATION OF THE EFFECT OF BOUNDARY REFLECTIONS

2.3.1 Wave reflection using soft ends at the boundary

In Dynamic Soil-Structure-Interaction (DSSI) model tests, the deformation of the soil medium is restricted by the artificial boundaries. Also, during seismic shaking, the soil near the boundaries may undergo compression and extension deformations causing the generation of P-waves. A good review on the different types of soil container can be found in Bhattacharya et al. (2011). Over the past decade, researchers have developed different types of model container to minimise the effects introduced by the artificial boundaries. One example is represented by flexible soil containers. Assuming that the soil layer and the adjacent end-walls behaves as an assembly of equivalent shear beams, the container is designed to mimic the shear beam response. This can be achieved by matching the shear stiffness between the model container and the soil it includes (Dar, 1993; Zeng & Schofield, 1996; Madabhushi et al., 1998; Bhattacharya et al., 2004; Elgamal et al., 2005; Pitilakis et al., 2008). This type of container is commonly made by aluminium rings spaced by soft rubber layers which provide the desired lateral stiffness. However, due to the high non-linearity of the soil, particularly at large strains, the matching
between the soil and container stiffness is possible only for a predefined range of strains, and it is generally not suitable for soil subjected to large deformations, such as those measured during soil liquefaction.

A new type of model container with flexible boundaries was first introduced by Whitman et al. (1981) to study liquefaction phenomena. This new container concept has been subsequently used by many research teams (Hushmand et al., 1988; Law et al., 1991; Elgamal et al., 1996; Brennan et al., 2006; Pamuk et al., 2007; Turan et al., 2009). The design principle of the flexible laminar box consists of minimising the lateral stiffness of the container to match the one of the liquefied soil column. This can be achieved by using a stack of aluminium rings supported individually with bearings, which permit a relative movement between the rings with minimal frictions. Several studies have confirmed that the laminar container is compatible with the large soil deformation expected during the simulation of earthquake-induced liquefaction. However, the laminar box may not replicate the actual boundary conditions when the soil column is not fully liquefied or is subjected to low strain levels.

The model container with rigid ends has been used by several research groups in both centrifuge (Whitman & Lambe, 1986; Adalier & Elgamal, 2002) and 1-g shaking table tests (Taguchi et al., 1992; Fishman et al., 1995; Lee & Santamarina, 2007). Numerical studies conducted by Whitman & Lambe (1986) and Fishman et al. (1995) have demonstrated that the effects caused by the rigid boundaries are significant up to a distance of 1.5-2.0 times the depth of the soil stratum. To increase the volume of soil subjected to the free-field condition, soft material can be placed on the inner sides of the model container, which in turns diminish the reflection of body waves from the boundaries and also the P-wave generation.

Duxseal material (a putty-like, pipe sealant rubber mixture compound) has been extensively used in the past decade for centrifuge modelling (Coe et al., 1985; Cheney et al., 1990; Cilingir & Madabhushi, 2011; Pak et al., 2011; Soudkhah & Pak, 2012) due to its high damping and relatively high stiffness required for high stress level attained during the spin-up process. Cilingir & Madabhushi (2011) reported that Duxseal can absorb up to 65% of incident P-waves. In experimental modelling conducted on a shaking table at normal gravity, the relatively low stress at which the model is subjected, makes Duxseal material too stiff and therefore it is generally
replaced by softer material such as conventional foam (Ha et al., 2011; Lombardi & Bhattacharya, 201X).

Recently, due to the relatively simple design and low cost of the material, the use of conventional foam as absorbing boundary for geotechnical model container has become an alternative solution to more expensive and complex soil containers. However, the foam-soil interaction and the amount of energy actually reduced by the conventional foam are still uncertain. Therefore the aim of this research was to investigate, through a series of shaking table tests, the dynamic performance of a rigid container having absorbing boundaries and to quantify the amount of energy absorbed by the conventional foam.

### 2.3.2 Experimental investigation

The tests were conducted using the shaking table available at the *Bristol Laboratory for Advanced Dynamics Engineering* (BLADE) at the University of Bristol (UK). The shaking table consists of a 3×3m cast aluminium platform driven by 8 servo hydraulic actuators, which allowed a full control of motion in all six-degrees of freedom. Each actuator had a dynamic capacity of 70 kN and maximum stroke of 300 mm, permitting to apply a maximum acceleration 1.6 and 1.2 g in the horizontal and vertical direction respectively (considering a payload of 10 tonnes). The table had an operational frequency range of 0-100 Hz.

The soil container was a rigid box with internal dimensions of 450 mm long, 200 mm wide and 400 mm high, see Figure 2.21. The container was formed by assembling 5 sheets of PTFE (Poly-Tetra-Fluoro-Ethylene) having a thickness of 25 mm, which were connected together by thread-locked bolts. To minimise the generation and reflection of the body waves from the rigid walls, panels of foam were placed on the inner sides of the end-walls, i.e. sides perpendicular to the direction of the shaking. The soil used in the tests was a relatively uniform dry layer of Red Hill 110 sand. Homogeneity of the soil deposit was achieved by using the dry pluviation technique.
The absorbing material comprised of commercially available conventional foams and the properties are shown in Figure 2.22 obtained from laboratory testing. The experimental campaign consisted of eight tests carried out on four different boundary arrangements, namely, no foam, foam 1, foam 2 and foam 3. Each configuration was subjected to broadband random white noise having 0.5-100 Hz bandwidth frequency. Moreover, to investigate the nonlinearity of the phenomenon, each configuration was subjected to two different levels of acceleration, i.e. 0.1 and 0.5 g. The acceleration measurements were recorded by 7 piezoelectric accelerometers with a frequency range between 0.5 and 3000 Hz. A schematic of the instrumentation layout is illustrated in Figure 2.23.
In this report, the results are presented by comparing the transmissibilities corresponding to different boundary arrangements, namely: no foam, foam 1, foam 2 and foam 3. Figure 2.24 shows the results calculated considering accelerometer ACC-2 as output and ACC-1 as input (for more details regarding the instrumentation layout see Figure 2.23).

Figure 2.24. Comparison of the transmissibilities relative to different boundary arrangements, estimated from signals recorded by accelerometers ACC-2 (output) and ACC-1 (input).
The maximum transmissibility corresponded to the response obtained from the container with rigid walls. On the contrary, the results computed when the absorbing foam layers were placed on the walls, showed a significant reduction in both magnitude and frequency content of the FFT.

For a theoretical semi-infinite medium subjected to one-dimensional shaking, the energy associated with the shear mode must be constant on the horizontal planes of the soil layer. However, in a model container, the presence of artificial boundaries may alter the horizontal energy distribution. Specifically, volumes of soil closer to the walls can be characterised by higher magnitudes and broader frequency contents due to generation and reflection of body waves from the edges. To investigate this phenomenon, the responses were evaluated considering as output the signal recorded by accelerometer ACC-3, which is located near the end-wall. The results are illustrated in Figure 2.25 and the following observations can be drawn.

- Firstly, the magnitude and frequency content of the transmissibilities relative to the rigid container did not vary significantly in comparison with the response estimated from accelerometer ACC-2 (shown in Figure 2.24).
- Secondly, the transmissibilities corresponding to the configurations with foams showed lower magnitudes especially for frequencies near the resonance of the soil deposit, which was about 60 Hz for the system having foams and about 65 Hz for the system with rigid walls.
- Thirdly, the magnitudes obtained for frequencies in the range between 30-55 Hz were higher than the ones computed from the rigid container. This discrepancy suggests that the effects due to the generation of P-waves and reflection of body waves from the boundaries affects the response of the system in the frequency content from 30 to 55 Hz.

Similar results were observed for other tests. Analyses of the test results suggest that models with foams had significantly lower energy and narrower frequency contents than those computed from the container with rigid boundaries.
Figure 2.25. Comparison of transmissibilities relative to different boundary arrangements, estimated from signals recorded by accelerometers ACC-3 (output) and ACC-1 (input).
3 APPLICATION OF FHT TECHNIQUE FOR SSI PROBLEMS AT NTUA

3.1 INTRODUCTION

This report presents the first attempt of the LEE – NTUA research team to perform RTDS tests with shaking table for SSI problems. To this, a novel framework was formulated on the basis of adaptive signal processing and parameter estimation methods. The former were utilized in order to compensate the dynamics of the shaking table, while the latter were used for the compensation of the total transfer delay. Specifically, an adaptive inverse control scheme was designed (in conjunction with task JRA1.2) and placed between the numerical substructure and the transfer system, which “cancels” the dynamics of the shaking table. It follows that the cascade of the adaptive controller and the shaking table becomes a delayed unit impulse response. To compensate this delay, a multistep ahead predictor was estimated by performing parametric and non–parametric identification on the specimen (physical substructure). Moreover, the applied methodology is characterized by two additional innovative features: (1) it replaces the traditional displacement command to the physical substructure by the acceleration one. This allows a wider response spectrum to be actually implemented to the specimen; and (2) it replaces the traditional load cell sensor for force feedback by an accelerometer that is placed on the specimen mass.

The method was initially applied to a simple, linear SDOF structure (physical model) on a horizontally deformable soil. Thus, the SSI problem pertains to two – DOF. This implies that only the linear part of the macroelement discussed in Section 1.2.3 is used and only in the horizontal dimension (that corresponds to stiffness $K\nu\nu$). Yet, to take dissipative effects into account, a damper was also added to the soil dynamics. Two specimens were tested, with periods of 0.5s and 0.2s, both having a specimen – to – foundation mass ratio equal to 4.
This section is organized as follows: in Section 3.2, a numerical formulation of the horizontal SSI problem is presented, by means of the two–DOF system, including substructuring and some initial simulations. Section 3.3 contains the applied methodology for RTDS, while Section 3.4 illustrates the experimental application. In Section 3.5, an alternative tracking controller for use in RTDS is introduced, that is based on a modified filtered–X algorithm. Finally, in Section 3.6 the results are summarized and issues for further development are proposed.

### 3.2 NUMERICAL FORMULATION OF THE HORIZONTAL SSI PROBLEM

#### 3.2.1 Equations of motion

Figure 3.1 illustrates a lumped–mass elastic specimen with a rigid footing of mass $m_b$. The specimen is modeled as a SDOF system of mass $m_1$ while the column is considered weightless and inextensible in the vertical axis, providing an elastic resistance to the motion of the relative mass, which is described by the stiffness $k_1$. In addition, a damper $c_1$ has been added, in order to provide a velocity–proportional resistance to the horizontal motion of the mass. The (horizontal) dynamics of the soil are described by the elastic stiffness $k_x$ and damping $c_x$. It is assumed that the foundation is supported on an elastic half–space and that the foundation dimensions are sufficiently small, so that the $\tau$–effects (Clough and Penzien 2003) are negligible.

![Figure 3.1. Lumped elastic structure on rigid foundation.](image)
By denoting as \( u_b(t) \) and \( u_1(t) \) the displacements of the foundation and the specimen, respectively, caused by the earthquake motion \( \ddot{x}_g(t) \), and by applying the equilibrium of forces to each mass, it follows that the corresponding two-DOF system is described by

\[
M \cdot \ddot{u}(t) + C \cdot \dot{u}(t) + K \cdot u(t) = -M \cdot r \cdot \ddot{x}_g(t) \tag{3.1}
\]

where \( u(t) = [u_b(t) u_1(t)]^T \), \( r = [1 \ 1]^T \) and

\[
M = \begin{bmatrix} m_b & 0 \\ 0 & m_1 \end{bmatrix}, \quad C = \begin{bmatrix} c_x + c_1 & -c_1 \\ -c_1 & c_1 \end{bmatrix}, \quad K = \begin{bmatrix} k_x + k_1 & -k_1 \\ -k_1 & k_1 \end{bmatrix} \tag{3.2}
\]

### 3.2.2 Substructuring

The substructuring of the structural system displayed in Figure 3.1 can be realized in various ways. Here, the lumped elastic specimen is chosen to serve as the physical substructure, while the foundation and the soil dynamics serve as the numerical substructure. The corresponding configuration is presented in Figure 3.2, where the interaction dynamics are also shown (quantities with red arrows). From Figure 3.2 it is clear that the original structural equation is not suitable for the purposes of substructuring. To this, Equation (3.1) must split in a way that renders the configuration of Figure 3.2. It is important to emphasize that, unlike current substructuring schemes, which utilize displacement as the output of the numerical model, the current one uses acceleration.

![Figure 3.2](image-url)

**Figure 3.2.** (a) Numerical substructure and (b) Physical substructure. The red arrows indicate the dynamics at the boundary.
Following the above discussion, the equation that describes the motion of the foundation can be expressed as

\[ m_b \cdot \ddot{u}_b(t) + c_x \cdot \dot{u}_b(t) + k_x \cdot u_b(t) = -m_b \cdot \ddot{x}_g(t) + F(t) \quad (3.3) \]

where

\[ F(t) = c_1 \cdot [\dot{u}_1(t) - \dot{u}_b(t)] + k_1 \cdot [u_1(t) - u_b(t)] \quad (3.4) \]

is the base shear force. For the specimen mass the corresponding equation is:

\[ m_1 \cdot \ddot{u}_1(t) = -m_1 \cdot \ddot{x}_g(t) - F(t) \quad (3.5) \]

\[ = -m_1 \cdot \ddot{x}_g(t) - c_1 \cdot [\dot{u}_1(t) - \dot{u}_b(t)] - k_1 \cdot [u_1(t) - u_b(t)] \]

Adding and subtracting the term \( m_1 \cdot \ddot{u}_b(t) \) from the left – hand side, Equation (3.5) becomes:

\[ m_1 \cdot [\ddot{u}_1(t) - \ddot{u}_b(t)] = -m_1 \cdot [\ddot{x}_g(t) + \ddot{u}_b(t)] - c_1 \cdot [\dot{u}_1(t) - \dot{u}_b(t)] - k_1 \cdot [u_1(t) - u_b(t)] \quad (3.6) \]

Since, in view of Figure 3.1, \( u_1(t) - u_b(t) = v_1(t) \), equal to the displacement of the specimen relative to the foundation mass, the last equation takes the form:

\[ m_1 \cdot \ddot{v}_1(t) + c_1 \cdot \dot{v}_1(t) + k_1 \cdot v_1(t) = -m_1 \cdot [\ddot{x}_g(t) + \ddot{u}_b(t)] \quad (3.7) \]

Equations (3.3), (3.4) and (3.7) describe fully the SSI substructuring problem. To show that this is indeed the case, observe that the base shear force can be expressed as:

\[ F(t) = c_1 \cdot \dot{v}_1(t) + k_1 \cdot v_1(t) \quad (3.8) \]

Using Equation (3.7), the right – hand side of Equation (3.8) can be written as follows:

\[ c_1 \cdot \dot{v}_1(t) + k_1 \cdot v_1(t) = -m_1 \cdot [\ddot{x}_g(t) + \ddot{u}_b(t) + \ddot{v}_1(t)] \quad (3.9) \]

Thus,

\[ F(t) = -m_1 \cdot [\ddot{x}_g(t) + \ddot{u}_b(t) + \ddot{v}_1(t)] \quad (3.10) \]
Equation (3.10) illustrates the fact that, in order to measure the base shear force it is not necessary to use a load cell. Knowing the mass of the physical substructure, this force can be measured by simply attaching an accelerometer to the specimen mass. This observation relaxes the need for load cell attachment between the physical substructure and the shaking table, which suffers from some serious drawbacks, including the addition of unnecessary dynamics into the system, space requirements and limitation of the effective measurement bandwidth.

In summary, the proposed substructuring algorithm for the case of the horizontal SSI problem is shown in Table 3.1. In every step, the shaking table is driven by the absolute acceleration of the foundation, which is calculated from the numerical model, as a result of the applied earthquake excitation and the resulted base shear force from the previous step. The latter is measured by an accelerometer attached to the mass of the specimen. The actual realization of this loop is discussed in Section 3.3. It is emphasized that this step is completed within one sample period (refer also to Section 1.2.1).

**Table 3.1. Steps for the substructuring of the horizontal SSI problem (see also Section 3.3).**

<table>
<thead>
<tr>
<th>Numerical substructure</th>
<th>[ m_b \cdot \ddot{u}_b(t) + c_x \cdot \dot{u}_b(t) + k_x \cdot u_b(t) = -m_b \cdot \ddot{x}_g(t) + F(t) ]</th>
</tr>
</thead>
<tbody>
<tr>
<td>Physical substructure</td>
<td>[ m_1 \cdot \ddot{v}_1(t) + c_1 \cdot \dot{v}_1(t) + k_1 \cdot v_1(t) = -m_1 \cdot [\ddot{x}_g(t) + \ddot{u}_b(t)] ]</td>
</tr>
<tr>
<td>Input to shaking table</td>
<td>[ \ddot{x}_g(t) + \ddot{u}_b(t) ]</td>
</tr>
<tr>
<td>Force measurement</td>
<td>[ F(t) = -m_1 \cdot [\ddot{x}_g(t) + \ddot{u}_b(t) + \dot{v}_1(t)] ]</td>
</tr>
</tbody>
</table>

### 3.2.3 Preliminary numerical analysis

In order to obtain an initial insight on the performance of the structural system under study, some preliminary numerical simulations were carried out, under two cases of earthquake excitation: sinusoidal acceleration and real earthquake acceleration record. Tables 3.2 and 3.3 show the values of the involved structural parameters and the resulted vibration modes, respectively. The structural system is characterized by specimen to foundation mass ratio equal to 4 and high soil stiffness that causes the natural frequencies to be spread in a wide range. In addition, the
specified damping values result in a damping matrix that is non-proportional and this is reflected to the mode shapes, which appear complex. This in turn implies that the relative position of each mass can be out of phase, by the amount indicated by the complex part of the mode shape.

Table 3.2. Parameters of the system used in the preliminary numerical analysis

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Soil stiffness</td>
<td>$k_x$</td>
<td>530000</td>
<td>$kN/m$</td>
</tr>
<tr>
<td>Soil damping</td>
<td>$c_x$</td>
<td>71.9036</td>
<td>$kN \cdot s/m$</td>
</tr>
<tr>
<td>Foundation mass</td>
<td>$m_p$</td>
<td>0.9755</td>
<td>$Mg$</td>
</tr>
<tr>
<td>Specimen mass</td>
<td>$m_1$</td>
<td>3.9020</td>
<td>$Mg$</td>
</tr>
<tr>
<td>Specimen stiffness</td>
<td>$k_1$</td>
<td>587.5600</td>
<td>$kN/m$</td>
</tr>
<tr>
<td>Specimen damping</td>
<td>$c_1$</td>
<td>2.5760</td>
<td>$kN \cdot s/m$</td>
</tr>
</tbody>
</table>

Table 3.3. Vibration modes of the system used in the preliminary numerical analysis

<table>
<thead>
<tr>
<th>Mode No.</th>
<th>Natural frequency (Hz)</th>
<th>Damping ratio (%)</th>
<th>Mode shape</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1.952</td>
<td>2.686</td>
<td>[1 $902.78 \pm 3.35i]^T$</td>
</tr>
<tr>
<td>2</td>
<td>117.378</td>
<td>5.176</td>
<td>[1 $-0.0003 \pm 0.0001i]^T$</td>
</tr>
</tbody>
</table>

For the simulations, the structural system of Equation (3.1) is transformed into a state space model of the form

$$\ddot{\xi}(t) = F \cdot \dot{\xi}(t) + G \cdot \ddot{x}_g(t)$$  \hspace{1cm} (3.11a)

$$y(t) = H \cdot \dot{\xi}(t) + D \cdot \ddot{x}_g(t)$$  \hspace{1cm} (3.11b)

where $\xi(t) = [u(t)\dot{u}(t)]^T$ is the state vector, $y(t) = [\ddot{u}_b(t)\ddot{u}_1(t)]^T$ is the output vector and

$$F = \begin{bmatrix} O_2 & I_2 \\ -M^{-1} \cdot K & -M^{-1} \cdot C \end{bmatrix}, \quad G = \begin{bmatrix} O_{2 \times 1} \\ -r \end{bmatrix}, \quad H = \begin{bmatrix} -M^{-1} \cdot K & -M^{-1} \cdot C \end{bmatrix}, \quad D = -r$$  \hspace{1cm} (3.12)
are the involved matrices of the state and the output equations, with $\mathbf{M}, \mathbf{C}, \mathbf{K}$ given by Equation (3.2) and $\mathbf{O}, \mathbf{I}$ denoting zero and unit matrices of indicated sizes, respectively. Simulations are performed using the Bogacki – Shampine method with variable step – size and relative tolerance 0.001. The simulated data are subsequently sampled at frequency $F_s = 1 \text{kHz}$.

Figure 3.3(a) shows the sinusoidal acceleration excitation of frequency $f = 2 \text{ Hz}$, and Figures 3.3(b) & (c) display the absolute accelerations of the foundation and the specimen, respectively. It is obvious that the high soil stiffness dominates the performance of the foundation, the absolute acceleration of which is almost identical to that of the excitation. On the contrary, the specimen exhibits very high relative acceleration (the absolute acceleration reaches up to 3g) that does not fade out even after 10 seconds of the earthquake excitation’s termination. This behaviour was expected, due the extremely lightly – damped nature of the first mode (see Table 3.3), which dominates the response of the specimen. Naturally, the base shear force climbs up to 150 kN. Notice that the excitation frequency is close to the natural frequency of the first mode.

The structural response of the system under a real seismic excitation record (Kalamata, 1986 earthquake at 50% scale, Figure 3.5(a)), is illustrated in Figures 3.5(b) & (c). The foundation exhibits similar performance, as in the previous case, while the relative acceleration of the specimen, as well as the base shear force (Figure 3.6) are significantly reduced in amplitude, due to the fact that the spectral content of the earthquake time – series is far from the resonant frequency. Yet, the vibration of the specimen after the termination of the earthquake still remains.
Shaking table test techniques and fault rupture box testing for SSI

Figure 3.3. (a) Sinusoidal acceleration excitation of frequency $f = 2\, Hz$; (b) Absolute acceleration of the foundation; (c) Absolute acceleration of the specimen.

Figure 3.4. Base shear force (sinusoidal excitation).
Shaking table test techniques and fault rupture box testing for SSI

Figure 3.5. (a) Kalamata earthquake (50% scale); (b) Absolute acceleration of the foundation; (c) Absolute acceleration of the specimen.

Figure 3.6. Base shear force (50% of Kalamata earthquake).
3.3 REAL – TIME DYNAMIC SUBSTRUCTURING OF THE HORIZONTAL SSI PROBLEM

The analysis of the previous Section provides a simple framework for the realization of the horizontal SSI problem using RTDS. Figure 3.7 displays a general configuration when the transfer system is a shaking table. The idea is to force the shaking table to replicate the output acceleration of the numerical model (absolute acceleration of the foundation). Yet, such a configuration is problematic, as the transfer system has its own dynamics, which interfere between the numerical and the physical substructure, causing the shaking table to produce an altered acceleration record.

![Basic configuration of RTDS with shaking table for the horizontal SSI problem.](image1)

Figure 3.7. Basic configuration of RTDS with shaking table for the horizontal SSI problem.

![Proposed configuration of RTDS with shaking table for the horizontal SSI problem.](image2)

Figure 3.8. Proposed configuration of RTDS with shaking table for the horizontal SSI problem.
In order to overcome the aforementioned problem, the advanced configuration, which is depicted in Figure 3.8, is proposed. Since the transfer system dynamics are not known a priori, they must be estimated, including the transfer delay, which must be compensated. To this, the theory of adaptive inverse control is utilized. Specifically, two steps are employed:

1. An adaptive identification framework is formulated, both off–line and on–line, and the dynamics of the transfer system are identified, including the transfer delay.
2. Following Step 1, adaptive inverse identification is performed and an inverse controller is identified and placed between the numerical structure and the transfer system.

As shown in Figure 3.8, the result of this two–step procedure is a system with delayed impulse response between the physical and the numerical substructure. This means that the output acceleration of the numerical structure is replicated without amplitude alteration, but with a delay of $\Delta$ – steps (refer to Section 3.2 for further details). In order to handle this effect, the following steps are integrated into the process:

1. Parametric identification is applied to the physical substructure and the structural dynamics of the specimen are estimated.
2. A $\Delta$ – step ahead predictor of the specimen’s absolute acceleration is formulated, on the basis of the identified model of the previous step.
3. Since the excitation time–series is known, the numerical model is configured to be executed at $\Delta$ – step ahead. The output of the numerical model at time $t + \Delta$, namely the absolute acceleration of the foundation $\ddot{x}_g(t + \Delta) + \ddot{u}_b(t + \Delta)$, is forwarded through the adaptive inverse controller and the transfer system, providing the acceleration $\ddot{x}_g(t) + \ddot{u}_b(t)$ to the shaking table.

In the following, details about each step of the proposed configuration are given.

### 3.3.1 Adaptive identification

The adaptive procedure of specifying the unknown transfer dynamics is illustrated in Figure 3.9. The (discrete – time) adaptive modeling system samples the input ($x[t]$) and the output ($y[t]$) of
the transfer system and adjusts its internal parameters, in order to produce a sampled output \( (\hat{y}[t]) \) (hat denotes estimator / estimate) which closely matches \( y[t] \). Once the adaptation has converged, the adaptive filter provides sufficient information about the transfer system dynamics. The adaptive identification procedure that is diagrammed in Figure 3.9 is a mature engineering field with diverse application areas such as control, channel equalization, geophysical signal processing, etc. (Glentis et al. 1998).

![Diagram](image)

**Figure 3.9. Adaptive identification of the transfer system (having an impulse response \( h[t] \)).** The adaptive filter adjusts its weights \( \hat{h}[t] \), so that its output matches the achieved acceleration of the shaking table.

Focusing on the adaptive identification of the transfer system, several issues arise, such as the choice of an appropriate input signal and the structure of the adaptive filter. Moreover, a critical issue which somehow distinguishes this specific application from conventional adaptive identification schemes is that the adaptation must be accomplished with the physical substructure installed on the shaking table. This necessity imposes further difficulty, since the installed specimen introduces a strong disturbance to the output of the transfer system. In fact, as the mass of the specimen approaches the mass of the shaking table, more disturbance is added to the output of the transfer system. This fact limits the range of adaptive algorithms that can be effectively applied to those that retain the ability of very fast convergence.
While several other alternatives are possible (and are currently being investigated by the LEE NTUA research team) the identification is set on the basis of LMS – FIR structures. The adaptive filter has the form:

\[
\hat{y}[t] = \sum_{k=0}^{n} \hat{h}[t, k] \cdot x[t - k] = \hat{h}^T[t] \cdot x[t]
\]  
(3.13)

where

\[
\hat{h}[t] = \left[ \hat{h}[t, 0] \hat{h}[t, 1] \ldots \hat{h}[t, n] \right]^T
\]  
(3.14)

\[
x[t] = \left[ x[t] \ x[t - 1] \ldots \ x[t - n] \right]
\]  
(3.15)

In Equation (3.13), \( \hat{h}[k] \) are the filter weights and \( n \) the order of the FIR structure. The update of the filter weights in every iteration is given by

\[
\hat{h}[t + 1] = \hat{h}[t] + \mu [t] \cdot g[t]
\]  
(3.16)

where \( \mu \) is the step size and \( g[t] \) a descent direction [16]. This direction is updated by a time–domain decorrelation method, proposed in [17]. Specifically, \( g[t] \) is obtained through the stochastic approximation

\[
g[t] = x[t] - a[t] \cdot x[t - 1]
\]  
(3.17)

where

\[
a[t] = \frac{x^T[t] \cdot x[t - 1]}{x^T[t-1] \cdot x[t-1]}
\]  
(3.18)

is the decorrelation coefficient between \( x[t] \) and \( x[t - 1] \) at time \( t \). The step size is calculated by

\[
\mu [t] = \frac{e[t]}{x^T[t] \cdot g[t]}
\]  
(3.19)

where \( e[t] = y[t] - \hat{y}[t] \).
The applied method for the adaptive identification of the transfer system belongs to a family of adaptive instrumental variable methods (Glentis et al. 1998) and is characterized by improved convergence properties and low computational complexity. Its steps are summarized in Table 3.4.

Table 3.4. Steps of the decorrelated LMS algorithm. Parameter $\rho$ is a trimming factor.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Calculate filter’s output</td>
<td>$\hat{y}[t] = \hat{h}^T[t] \cdot x[t]$</td>
</tr>
<tr>
<td>2</td>
<td>Calculate error</td>
<td>$e[t] = y[t] - \hat{y}[t]$</td>
</tr>
<tr>
<td>3</td>
<td>Calculate decorrelation coefficient</td>
<td>$a[t] = \frac{x^T[t] \cdot x[t-1]}{x^T[t-1] \cdot x[t-1]}$</td>
</tr>
<tr>
<td>4</td>
<td>Update gradient</td>
<td>$g[t] = x[t] - a[t] \cdot x[t-1]$</td>
</tr>
<tr>
<td>5</td>
<td>Update step size</td>
<td>$\mu[t] = \rho \cdot \frac{e[t]}{x^T[t] \cdot g[t]}$</td>
</tr>
<tr>
<td>6</td>
<td>Update filter’s weights</td>
<td>$\hat{h}[t + 1] = \hat{h}[t] + \mu[t] \cdot g[t]$</td>
</tr>
</tbody>
</table>

The application of the decorrelated LMS algorithm to the identification of the transfer system leads to the availability of a mathematical description which describes the unknown dynamics. Normally, any delay terms of the transfer system are estimated as leading zeros in the weights of the adaptive filter. However, due to the presence of disturbance from the specimen, of unavoidable instrumentation noise, as well as of non–Gaussian excitation, these leading terms may not converge to zero at all. To this, an off–line procedure is additionally employed for the verification of transfer delay, which applies a Gaussian white noise acceleration to the shaking table and performs two distinct non–parametric tests for the verification of the delay. The former test estimates the sample cross–correlation function between the command and the achieved acceleration of the table, while the latter uses the same data to estimate the impulse response of the transfer system from the inverse discrete Fourier transform (DFT) of the frequency response function (FRF).

In detail, assume that the transfer system is described by an FIR model of the form of Equation (3.13). Then
where \( x[t] \) and \( y[k] \) are the reference and the achieved accelerations of the transfer system, respectively, and \( \tau_1 \) is the transfer delay. Their cross–correlation function is given by

\[
R_{xy}[\tau] = E\{x[t] \cdot y[t + \tau]\} \tag{3.21}
\]

with \( \tau \) denoting the time lag. Substituting Equation (3.20) to Equation (3.21) yields

\[
R_{xy}[\tau] = E \left\{ x[t] \cdot \sum_{k=0}^{n} h[k] \cdot x[t + \tau - k - \tau_1] \right\}
\]

\[
= E \left\{ \sum_{k=0}^{n} h[k] \cdot x[t] \cdot x[t + \tau - k - \tau_1] \right\} \tag{3.22}
\]

\[
= \sum_{k=0}^{n} h[k] \cdot E\{x[t] \cdot x[t + \tau - k - \tau_1]\}
\]

where \( R_{xx}[\cdot] \) is the autocorrelation of the reference signal. When the latter is Gaussian white noise, at least up to the Nyquist frequency, its autocorrelation is given by

\[
R_{xx}[\tau] = \sigma_{xx}^2 \cdot \delta[\tau] \tag{3.23}
\]

where \( \sigma_{xx}^2 \) is the variance and Kronecker’s Delta function. Substitution in Equation (3.22) gives:

\[
R_{xy}[\tau] = \sigma_{xx}^2 \cdot \sum_{k=0}^{n} h[k] \cdot \delta[\tau - k - \tau_1] \tag{3.24}
\]

Applying the discrete–time equivalent of the shifting theorem, Equation (3.24) becomes:

\[
R_{xy}[\tau] = \sigma_{xx}^2 \cdot h[\tau - \tau_1] \tag{3.25}
\]

Since \( h[k] \) is zero outside the \([0 \ n]\) interval, the cross–correlation \( R_{xy}[\tau] \) is zero outside the \([\tau_1 \ n + \tau_1]\) interval. Thus, by estimating the sample cross–covariance function between the
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reference and the achieved acceleration, it is possible to obtain reasonable estimates for the transfer delay.

Remark 1: The previous analysis is valid only when the reference acceleration consists of an unfiltered Gaussian white noise time – series. In any other case, Equation (3.23) does not hold, so the cross – correlations $R_{xy}[\tau]$ are not zero outside the prescribed interval. This includes the case of bandlimited white noise, the autocorrelation of which is given by

$$R_{xx}[\tau] = a \cdot B \cdot \frac{\sin(2 \cdot \pi \cdot B \cdot \tau)}{2 \cdot \pi \cdot B \cdot \tau} \cdot \cos(2 \cdot \pi \cdot f \cdot \tau)$$  \hspace{1cm} (3.26)

where $B$ is the frequency band. Substituting Equation (3.26) into Equation (3.22) implies entirely different results.

Remark 2: In practice, the following steps are utilized for the estimation of the transfer delay using the cross – correlation approach:

1. Excite the transfer system with Gaussian white noise acceleration (reference signal) and sample the achieved acceleration of the shaking table.
2. Estimate the normalized sample cross – correlation function $\bar{R}_{xy}[\tau]$ for $\tau = 0, ..., N/4$, where $N$ is the length of the data, as

$$\bar{R}_{xy}[\tau] = \frac{R_{xy}[\tau]}{\sqrt{R_{xx}[0] \cdot R_{yy}[0]}}$$  \hspace{1cm} (3.27)

3. Under the assumption of normality, it can be shown [18] that $\bar{R}_{xy}[\tau]$ can be considered zero, at 95% level of significance, when is contained within the $\pm 1.96/\sqrt{N}$bounds.
4. Estimate the transfer delay as the leading values (normally $\tau_1$) that fall within the bounds of Step 3, before a non – zero one appears.
5. If required, combine Equations (3.25), (3.27) and obtain a rough estimate of $h[t]$ as

$$\hat{h}[\tau] = \frac{\sigma_{xx}}{\sigma_{yy}} \cdot \bar{R}_{xy}[\tau]$$  \hspace{1cm} (3.28)

As noted above, the impulse response of the transfer system can be alternatively estimated by the inverse DFT of the estimated FRF:
\[
\hat{f}[f] = \frac{S_{xy}(f)}{S_{xx}(f)}
\]  

where \(S_{xy}(f)\) and \(S_{xx}(f)\) are the cross – spectral and autospectral densities of the indicated quantities, respectively. Both \(S_{xy}(f)\) and \(S_{xx}(f)\) can be estimated using ensemble averaging techniques that consist of splitting the series into overlapping segments, taking the DFT of every segment and calculating its (cross/auto) spectral density, and averaging the results. In order to avoid circularly biased estimates (Bendat and Piersol 2000), it is recommended to use the two – sided estimates of \(S_{xy}(f)\) and \(S_{xx}(f)\) and to avoid using tapering. The obtained impulse response estimate provides additional insight about the delay of the transfer system.

### 3.3.2 Adaptive inverse identification

The impulse response of the adaptive inverse controller of Figure 3.8 is just the reciprocal of the one that describes the transfer system (including its delay). Yet, since the transfer system is dominated by internal delay, the inverse controller may have difficulty in overcoming it, since it must be a predictor. In addition, if the transfer system is nonminimum phase (transfer function zeros in the right half of the s-plane or outside the unit circle in the z-plane), then the inverse controller results unstable. Moreover, the disturbance of the transfer system biases the converged solution and prevents the formation of a proper inverse (Widrow and Wallach 2007).

To cope with these problems, two configurations were employed for the adaptive inverse identification of the transfer system, which are shown in Figure 3.10. The configuration of Figure 3.10(a) adapts the weight of the inverse model on – line, whereas the one of Figure 3.10(b) performs the same operation off – line. The critical point in both configurations is the use of the transfer system’s adaptive estimate to the estimation of the inverse. This is justified by the fact that the transfer system estimate has essentially the same dynamic performance as the plant but is free of disturbance. Therefore, a closer approximation to the desired delayed inverse can be obtained, which is unbiased from plant disturbance.
Figure 3.10. (a) On – line adaptive inverse identification of the transfer system; (b) Off – line adaptive inverse identification of the transfer system.
The main disadvantage of the configuration of Figure 3.10(a) is that the convergence of the on–
line adaptive algorithm may be very slow, especially when conventional LMS algorithms are
utilized, in order to cope with the high disturbance levels that appear to the transfer system due to
the presence of the specimen (Widrow and Wallach 2007). To cope with this problem, the
decorrelated LMS algorithm described in Section 3.1 is also applied to the inverse identification
process, causing a dramatic decrease in the convergence rate. Alternatively, having an estimate
of the transfer system’s impulse response available, the configuration of Figure 3.10(b) can be
applied. Instead of adapting the weights of the inverse controller on–line, the off–line process
of Figure 3.10(b) utilizes the same functionality. It consists of the on–line scheme of Figure 3.9
that estimates the impulse response of the transfer system, followed by an off–line simulation
that realizes a copy of that estimate to the estimation of the inverse controller. This configuration
can be run much faster than real time and, at the same time, it permits the selection of a properly
set simulation environment, in which several critical adaptation parameters can be tested and
controlled, such as the choice of the input signal, the length of the inverse model and the step
size.

The successful estimation of the transfer system and the adaptive inverse controller is a critical
step towards the design of the process depicted in Figure 3.8 and leads to a cascaded system with
impulse response of the form:

\[ g[t] = \delta[t - \Delta] \]  

(3.30)

where \( \Delta \) is the total delay of the cascade of the transfer system and the inverse controller.

3.3.3 Closing the loop: discretization, specimen identification and predictor
design

The successful functionality of the substructuring steps shown in Table 3.1, by means of the
process illustrated in Figure 3.8, requires some further steps. Since the realization of the whole
process must take place in discrete – time, the equation that describes that numerical substructure
must be discretized. Taking under consideration the nature of feedback (specimen’s absolute
acceleration), this equation must be first transformed into a more suitable form prior to
discretization. In view of Table 3.1, the substitution of the equation that describes the feedback force into the one that describes the structural equation yields:

$$m_b \cdot \ddot{u}_b(t) + c_x \cdot \dot{u}_b(t) + k_x \cdot u_b(t) = -m_b \cdot \ddot{x}_g(t) - m_1 \cdot [\ddot{x}_g(t) + \dot{u}_b(t) + \dot{v}_1(t)] \quad (3.31)$$

Simple algebraic manipulation transforms Equation (3.31) to:

$$\ddot{u}_b(t) + \frac{c_x}{m_b + m_1} \cdot \dot{u}_b(t) + \frac{k_x}{m_b + m_1} \cdot u_b(t) = -\ddot{x}_g(t) - \frac{m_1}{m_b + m_1} \cdot \dot{v}_1(t) \quad (3.32)$$

Equation (3.32) reveals that the relative acceleration of the foundation depends on the earthquake acceleration and the relative acceleration of the specimen. Furthermore, the contribution of the latter quantity is always smaller to that of the former, due to the ratio involved in the right side of Equation (3.32).

The discretization of Equation (3.32) is carried out by applying the bilinear (Tustin) approximation to the associated transfer function. More specifically, set

$$a_1 = \frac{c_x}{m_b + m_1}, \quad a_2 = \frac{k_x}{m_b + m_1}, \quad \beta_1 = \frac{m_1}{m_b + m_1} \quad (3.33)$$

and

$$a_g(t) = \ddot{x}_g(t), \quad a_v(t) = \dot{v}_1(t) \quad (3.34)$$

so that

$$\ddot{u}_b(t) + a_1 \cdot \dot{u}_b(t) + a_2 \cdot u_b(t) = -a_g(t) - \beta_1 \cdot a_v(t) \quad (3.35)$$

Applying the Laplace transform to Equation (3.35), assuming zero initial conditions, and multiplying by $s^2$, the following expression is derived for the relative acceleration of the foundation:

$$s^2 U_b(s) = -\frac{s^2}{s^2 + a_1 \cdot s + a_2} \cdot A_g(s) - \frac{\beta_1 \cdot s^2}{s^2 + a_1 \cdot s + a_2} \cdot A_v(s) \quad (3.36)$$
where \( U_b(s), \ A_b(s) \) and \( A_v(s) \) are the Laplace transforms of \( u_b(t), \ a_g(t) \) and \( a_v(t) \), respectively. Thus, two transfer functions are involved, namely:

\[
H_1(s) = \frac{s^2}{s^2 + a_1 \cdot s + a_2} \tag{3.37a}
\]

\[
H_2(s) = \frac{\beta_1 \cdot s^2}{s^2 + a_1 \cdot s + a_2} \tag{3.37b}
\]

The Tustin approximation is:

\[
s \rightarrow 2 \cdot F_s \frac{1 - z^{-1}}{1 + z^{-1}} \tag{3.38}
\]

with \( F_s \text{Hz} \) denoting the sampling frequency (the one at which the RTDS is going to be executed). Substituting Equation (3.38) into Equations (3.37) and performing a sequence of algebraic operations, the discrete – time transfer functions result to:

\[
H_1(z) = \frac{B_1 - 2 \cdot B_1 \cdot z^{-1} + B_1 \cdot z^{-2}}{1 + A_1 \cdot z^{-1} + A_2 \cdot z^{-2}} \tag{3.39a}
\]

\[
H_2(z) = \frac{B_2 - 2 \cdot B_2 \cdot z^{-1} + B_2 \cdot z^{-2}}{1 + A_1 \cdot z^{-1} + A_2 \cdot z^{-2}} \tag{3.39b}
\]

where

\[
A_1 = \frac{-8 \cdot F_s^2 + 2 \cdot a_2}{4 \cdot F_s^2 + 2 \cdot F_s \cdot a_1 + a_2}, \quad A_2 = \frac{4 \cdot F_s^2 - 2 \cdot F_s \cdot a_1 + a_2}{4 \cdot F_s^2 + 2 \cdot F_s \cdot a_1 + a_2} \tag{3.40}
\]

\[
B_1 = \frac{4 \cdot F_s^2}{4 \cdot F_s^2 + 2 \cdot F_s \cdot a_1 + a_2}, \quad B_2 = \frac{4 \cdot \beta_1 \cdot F_s^2}{4 \cdot F_s^2 + 2 \cdot F_s \cdot a_1 + a_2} \tag{3.41}
\]

Putting it all together, it follows that the discrete – time equivalent of Equation (3.32) for the relative acceleration case is given by:
where \(a_b[t]\) is the relative acceleration of the foundation and \(t\) represent the discrete – time unit, i.e. \(t = k \cdot T_s, \ k = 0,1, \ldots, N, \ T_s = 1/F_s\). Since in RTDS tests the sampling frequency is many times the natural frequencies of the structural system under study, Equation (3.42) is a good approximation of the numerical substructure, as long as the latter is linear.

In Table 3.1 it was noted that the acceleration which must be supplied to the shaking table at time \(t\) is equal to \(a_b[t] + a_g[t]\). Yet, the transfer system and the inverse controller cause a total delay of \(\Delta\) time steps, as Equation (3.30) indicates. To compensate this serious effect, Equation (3.42) must be shifted \(\Delta\) time steps forward, that is:

\[
a_b[t + \Delta] + A_1 \cdot a_b[t + \Delta - 1] + A_2 \cdot a_b[t + \Delta - 2] = -B_1 \cdot \{a_g[t + \Delta] - 2 \cdot a_g[t + \Delta - 1] + a_g[t + \Delta - 2]\}
\]

\[
- B_2 \cdot \{a_v[t + \Delta] - 2 \cdot a_v[t + \Delta - 1] + a_v[t + \Delta - 2]\}
\]

The problem of Equation (3.43) lies on the second term of the right hand side: the relative acceleration of the specimen is unknown at the involved time steps. To deal with this situation, a predictor is formulated. In what follows, the idea is applied for the case of the linear specimen described in Section 3.2, but it can be extended to specimens with many degrees of freedom and even to time varying and nonlinear structures.

The specimen is described by the equation

\[
m_1 \cdot \ddot{v}_1(t) + c_1 \cdot \dot{v}_1(t) + k_1 \cdot v_1(t) = -m_1 \cdot [\ddot{x}_g(t) + \ddot{u}_b(t)]
\]

so the first step is to estimate the structural parameters. Usually, the specimen mass can be determined prior to the RTDS tests. Regarding the stiffness and the damping, these can be identified by applying either
1. a non-parametric method by evaluating the transmissibility function; or
2. a parametric method by means of a black-box or subspace methods by evaluating zeros and poles. (Ljung 1999).

Having estimated the natural frequency and the damping ratio, the stiffness and the damping of the structure are extracted by:

$$\hat{k}_1 = 4 \cdot \pi^2 \cdot f_1^2 \cdot m_1$$  \hspace{1cm} (3.45)
$$\hat{c}_1 = 4 \cdot \pi \cdot f_1 \cdot \zeta_1 \cdot m_1$$  \hspace{1cm} (3.46)

where $f_1$ and $\zeta_1$ are the natural frequency (in Hz) and the damping ratio, respectively. Notice that Equation (3.44) can also be written as

$$\ddot{v}_1(t) + 2 \cdot \zeta_1 \cdot \omega_1 \cdot \dot{v}_1(t) + \omega_1^2 \cdot v_1(t) = -[\ddot{x}_g(t) + \ddot{u}_b(t)]$$  \hspace{1cm} (3.47)

with $\omega_1 = 2 \cdot \pi \cdot f_1$, thus the step implied by Equations (3.45) – (3.46) can be skipped. Following the same discretization procedure, as in the case of the numerical substructure, the transfer function that expresses the acceleration of Equation (3.47) in discrete-time is:

$$G(z) = \frac{L_1 - 2 \cdot L_1 \cdot z^{-1} + L_1 \cdot z^{-2}}{1 + K_1 \cdot z^{-1} + K_2 \cdot z^{-2}}$$  \hspace{1cm} (3.48)

with

$$K_1 = \frac{-8 \cdot F_s^2 + 2 \cdot g_2}{4 \cdot F_s^2 + 2 \cdot F_s \cdot g_1 + g_2}, \hspace{1cm} K_2 = \frac{4 \cdot F_s^2 - 2 \cdot F_s \cdot g_1 + g_2}{4 \cdot F_s^2 + 2 \cdot F_s \cdot g_1 + g_2}$$  \hspace{1cm} (3.49)

$$L_1 = \frac{4 \cdot F_s^2}{4 \cdot F_s^2 + 2 \cdot F_s \cdot g_1 + g_2}$$  \hspace{1cm} (3.50)

and

$$g_1 = 2 \cdot \zeta_1 \cdot \omega_1, \hspace{1cm} g_2 = \omega_1^2$$  \hspace{1cm} (3.51)

The corresponding difference equation is (refer to Equation (3.34))
The $Z$-transform of Equation (3.52) is:

$$A_v(z) = \frac{-G(z) \cdot A_g(z) - G(z) \cdot A_v(z)}{A_{vg}(z)}$$  \hspace{1cm} (3.53)

Thus, in view of Equations (3.52) – (3.53), the response of $a_v[t]$ can be split into two parts: one due to the earthquake excitation (acceleration $a_{gl}[t]$) and one due to the relative acceleration of the foundation ($a_{vb}[t]$):

$$a_v[t] = a_{vg}[t] + a_{vb}[t]$$  \hspace{1cm} (3.54)

while the following equations hold

$$a_{vg}[t] + K_1 \cdot a_{vg}[t - 1] + A_2 \cdot a_{vg}[t - 2]$$
$$= -L_1 \cdot \{a_g[t] - 2 \cdot a_g[t - 1] + a_g[t - 2]\}$$  \hspace{1cm} (3.55)

$$a_{vb}[t] + K_1 \cdot a_{vb}[t - 1] + A_2 \cdot a_{vb}[t - 2]$$
$$= -L_1 \cdot \{a_b[t] - 2 \cdot a_b[t - 1] + a_b[t - 2]\}$$  \hspace{1cm} (3.56)

From Equation (3.55), it is obvious that $a_{vg}[t]$ is perfectly predictable, so the term $a_{vg}[t + \Delta]$ is indeed available. On the contrary, $a_{vb}[t + \Delta]$ can’t be available, since it depends on the relative acceleration of the foundation at the same time instant. To this, a predictor of this quantity is formulated, under the assumption that $a_{vb}[t]$ can be expressed according to Wold decomposition theorem (Box et al. 2008) as

$$a_{vb}[t] = G(z) \cdot e[t] = \sum_{k=1}^{\infty} g_{vg}[k] \cdot e[t - k]$$  \hspace{1cm} (3.57)
where $\left[t\right]$ is Gaussian white noise with zero mean and unit variance and $g_{v_b}[k]$ is the impulse response of $G(z)$. The approximation of Equation (3.57) can be used to design a $\Delta$ steps ahead predictor. Using the method described by Ljung (1999), it follows that

$$\hat{a}_{vb}[t + \Delta] + K_1 \cdot \hat{a}_{vb}[t + \Delta - 1] + A_2 \cdot \hat{a}_{vb}[t + \Delta - 2] = \sum_{k=0}^{nd} d_k \cdot a_{vb}[t - k]$$

(3.58)

where

$$D(z) = \sum_{k=0}^{nd} d_k \cdot z^{-k} = d_0 + d_1 \cdot z^{-1} + \cdots + d_{nd} \cdot z^{-nd}$$

(3.59)

is a polynomial that is calculated from the identity

$$K(z) = F(z) \cdot L(z) + z^{-\Delta} \cdot D(z)$$

(3.60)

In Equation (3.60), $K(z)$ and $L(z)$ are the numerator and denominator polynomials of $G(z)$, respectively, and $F(z)$ is a polynomial of order $\Delta - 1$ and coefficients equal to the first $\Delta$ terms of $g_{v_b}[k]$.

3.4 APPLICATION

3.4.1 Specimen description and experiment design

The method analyzed in the previous Section was applied to the horizontal SSI problem of the structure that pertains to Section 3.2. Figure 3.11 (a) displays the specimen that is used for the RTDS tests. Span length and height limitations imposed by the shaking table dimensions (4.00 m x 4.00 m), as well as force and overturning moment capacity, necessitated the construction of only one half of a frame. In specific, the specimen consists of a fixed – base column that is interconnected to a beam by a hinge. The boundary conditions applied to the free end of the beam pertain to free rotation around the horizontal axis and free sliding in the horizontal direction. This is achieved using a special device that provides a sliding pinned support. Under this configuration, the specimen is characterized by a single DOF. A sketch of the specimen is shown in Figure 3.11(b).
Figure 3.11. (a) The specimen that was used for the RTDS tests. (b) Sketch of the test specimen.
Both the column and the beam have a HEB240 cross section of grade S275, causing the self-weight of the specimen to be 1.102 Mgr, including the roller. In order to realize two distinct physical models, two different orientations of the column are considered. In the first (flexible version), the weak axis of the column is placed parallel to the X axis of the shaking table and a lumped mass of 2.35 Mgr is distributed on the beam. In the second (stiff version), the column is rotated by 90 degrees and a mass of 0.45 Mgr is applied to the span. The resulted physical models have periods at 0.5 s (2 Hz) and 0.2 s (5 Hz), respectively. It must be stretched out that the conduction of initial tests after specimen installation revealed a strong alteration of the shaking table’s performance, as a result of specimen’s response in both configurations. This is attributed to their mass (comparable to the mass of the shaking table) and geometry, which cause serious affection to the behaviour of the shaking table. To this, the corresponding RTDS tests prove extremely demanding, as a lot of effort must be paid to the compensation of any undesired performance.

RTDS tests are conducted using a National Instruments RT – Desktop controller, equipped with two DAQ cards. The numerical models have been designed at MATLAB / SIMULINK and subsequently transferred to LABVIEW using the Simulation Interface Toolkit. In any case reported below, the sampling frequency is $F_s = 1 \text{ kHz}$. The required feedback from the specimen is accomplished by attaching an accelerometer to the beam, right above the hinge.

### 3.4.2 Results for the specimen of period $T = 0.5 s$

Figure 3.12 shows the achieved absolute acceleration of the specimen under a four – octave logarithmic sine sweep test (frequency range at[1 16] Hz). There’s a clear pick around105 s, followed by a smaller one at around112 s. The corresponding FRF of the specimen, estimated using the recorded acceleration of the shaking table and the specimen acceleration of Figure 3.12, is shown at Figure 3.13 for the same frequency range. Estimation is based on Equation (3.29) using Welch’s method with Hanning windowing (number of FFT points$N_{FFT} = 2^{14}$, 50% overlapping). There’s a clear peak at the area of 2 Hz, at which the phase changes from zero to almost$\pi$ rad s. Using the peak picking method (PPM), the natural frequency and the damping ratio of the specimen are estimated $f_n = 1.953 \text{ Hz}$ and$\zeta_n = 2.690 \%$ , respectively. The natural frequency is very close to its theoretical counterpart, while the lightly damped nature of the
specimen is confirmed. Using the estimated modal values, the corresponding stiffness and
damping of the single DOF structure are found to be $\hat{K}_1 = 587.56 \text{kN/m}$ and $\hat{c}_1 = 2.576 \text{kN} \cdot \text{s/m}$, respectively, using Equations (3.45) and (3.46).

![Figure 3.12. Achieved acceleration of the shaking table at the sine sweep test (specimen of period $T=0.5$ s).](image1)

![Figure 3.13. Estimated FRF of the specimen of period $T = 0.5$ s using the sine sweep test data.](image2)
Figure 3.14. (a) Gaussian white acceleration excitation \((r_{ms} = 0.014 \, g)\) and (b) shaking table achieved absolute acceleration (specimen of period \(T=0.5s\)).

Figure 3.15. Impulse response of the transfer system (off-line), (a) normalized sample cross – correlation estimate, (b) inverse DFT of the FRF estimate and (c) decorrelated LMS estimate (specimen of period \(T=0.5s\)).
In order to gain a first insight about the transfer system, a test with Gaussian white noise acceleration of $rms = 0.014\ g$ (Figure 3.14(a)) is conducted and the absolute acceleration of the shaking table during the test is shown at Figure 3.14(b). It is emphasized that the excitation signal is unfiltered, which means that it is characterized by flat spectrum over the entire frequency range considered. Based on these data, Figure 3.15(a) – (c) displays the impulse response of the transfer system, estimated using the normalized sample cross – correlation function, the inverse DFT of the FRF and the decorrelated LMS algorithm (off – line), respectively, as discussed in Section 3.3.1. All three estimates are characterized by a damped oscillating behavior, as a result of the specimen’s presence. While the inverse FRF estimate (Figure 3.15(b)) has not identified any lag, the other two estimates have done so and this is very evident in Figure 3.15(c), where the decorrelated LMS filter has four trailing “zeros”. Moreover, as expected, all estimates are nonminimum phase, which means that their direct inverses are unstable and cannot be realized as inverse controllers. Thus, at a first glance, the impulse response of the transfer system is a nonminimum phase system with delay.

Having in mind this significant observation, an FIR model of the transfer system is identified on – line using the scheme of Figure 3.9. Various FIR models are tested, using Gaussian white noise excitation of variance $\sigma_{xx}^2 = 0.05\ g$ as input, both unfiltered and filtered with cut – off frequency equal to the Nyquist one ($f_s/2 = 500\ Hz$). It is noted that, where applicable, the same filter (direct – form FIR transposed of 40th order) is applied to the measured shaking table acceleration. Figure 3.16 shows the estimated impulse responses for three distinct tests. The first two impulse responses (Figure 3.16(a) – (b), filter lengths 500 and 1000, respectively) retain the same form as the off – line estimate presented in Figure 3.15(c) and this is natural, since these two estimates where obtained using unfiltered input – output data. In addition, it seems that an order of 500 taps is sufficient, as higher weights do not seem to contribute significant information. On the contrary, the impulse response of Figure 3.16(c) has the same damped oscillating structure, but differs substantially both in the leading and the trailing terms (observe that delay has vanished). This is the result of the filtering action to the data. However, during the subsequent tests for the identification of the inverse controller, this estimate presented the best performance among the three and it was thus chosen as the FIR filter that describes the transfer system. It is noted that the selected FIR filter is also nonminimum phase.
Shaking table test techniques and fault rupture box testing for SSI

Figure 3.16. Impulse response of the transfer system (on-line). (a) unfiltered data, FIR order 500, (b) unfiltered data, FIR order 1000 and (c) filtered data, FIR order 500 (specimen of period $T=0.5s$).

Due to time and safety reasons, the tests for the identification of the inverse controller are carried out off-line, by realizing the scheme of Figure 3.10(b). Two delay cases are investigated, namely a small delay of $\Delta = 10$ and a large delay $\Delta = 250$, with the latter corresponding to half the FIR filter order of the transfer system. Figure 3.17 presents the results of the adaptation process. It is obvious that the controller with the large delay has many leading terms close to zero, which can be discarded, leading to a controller with much less delay, as the one shown in Figure 3.17(a). However, using the small delay controller in cascade with the transfer system led to insufficient behavior and this can be justified from Figure 3.18, where the convolution of these two controllers with the estimated FIR model of the transfer system is displayed. It is obvious that the convolution of the large delay controller with the estimated FIR model of the transfer system leads to an impulse that is close to unity.
Figure 3.17. Impulse response of the inverse controller. (a) order 500, delay $\Delta = 10$ and (b) order 500, delay $\Delta = 250$ (specimen of period $T=0.5s$).

Figure 3.18. Convolution of the estimated FIR model of the transfer system (Figure 3.16(c)) and the inverse controllers of Figure 3.17: (a) low delay controller and (b) high delay controller (specimen of period $T=0.5s$).

Having estimated the inverse controller and the total delay of its cascade with the transfer system ($\Delta = 250$), the next step is to perform a waveform replication test using the excitations to be used RTDS. Figures 3.19 – 3.20 illustrate the results of these tests. In the former (Figure 3.19), a pure sinusoidal acceleration of amplitude 0.1 g and frequency 2 Hz is applied to the system,
forwarded at 250 time steps, and the achieved acceleration of the shaking table is recorded. As Figure 3.19(a) indicates, the matching of the two signals (reference and achieved) is quite good, except some sharp spikes which are observed at the achieved acceleration. These spikes are the result of the inability of the current system to apply analog filtering and smoothing at the output of the RT – Desktop controller, an issue that is currently being handled by the LEE – NTUA research team. Even so, the synchronization plot of Figure 3.19(b) shows that the satisfying degree of amplitudes between the two signals, as well as the compensation of the delay.

![Graph](image)

**Figure 3.19.** Results of the waveform replication test for sinusoidal input. (a) reference (black) and achieved (red) accelerations and (b) synchronization plot (specimen of period T=0.5s).

The procedure is repeated for the Kalamata earthquake acceleration case (50% scale) and the results are depicted in Figure 3.20. Again, delay compensation is successful, yet, the amplitude of the achieved acceleration does not coincide to that of the reference and this is clearly reflected in the synchronization plot of Figure 3.20(b). This observation necessitates the implementation of additional measures at the adaptation process. Such a measure that is currently being investigated is the realization of automatic gain controllers at both the transfer system estimation and the inverse controller adaptation stages.
Figure 3.20. Results of the waveform replication test for the Kalamata earthquake (50% scale). (a) reference (black) and achieved accelerations and (b) synchronization plot (specimen of period $T=0.5s$).

Figure 3.21. Results of the RTDS test for the sinusoidal input. (a) theoretical (black) and achieved (red) absolute acceleration of the foundation and (b) synchronization plot (specimen of period $T=0.5s$).
Figure 3.22. Results of the RTDS test for the sinusoidal input. Theoretical (black) and achieved (red) base shear force (specimen period $T=0.5\text{s}$).

With the above mismatches detected and analyzed, two RTDS tests are now performed for the two excitation cases considered, using the soil dynamics and the foundation mass shown in Table 3.1, by realizing the process of Figure 3.8. Figures 3.21 – 3.22 display the results for the sinusoidal excitation.

Clearly, the absolute acceleration of foundation is close to its theoretical counterpart, while, on the contrary, the base shear forces result close, yet with notable differences. This inconsistency can be justified by taking a closer look at the specimen acceleration of the sine sweep test shown in Figure 3.12: observe that the first resonance dies out very quickly (i.e. it’s “asymmetric”), which maybe indicates the presence of a highly damped mode, while the second resonance has not been taken into account at all. Yet, this issue requires further investigation. Recall also that the frequency of the excitation is very close to the first resonant point. Perhaps that is why the base shear force is amplified at the transient part of the test, as shown in Figure 3.22.

The results for the Kalamata case are presented in Figures 3.23 – 3.24, where similar remarks are observed. The absolute acceleration of the foundation is close to its theoretical counterpart, while the base shear force retains similar amplitude to the theoretical one. The amplitude mismatches are due to the imperfect impulse response of Figure 3.18. Introducing an automatic gain controller may improve this inconsistency.
3.4.3 Results for the specimen of period $T = 0.2s$

A similar process is realized in the case where the column of the specimen is rotated by 90 degrees about its vertical axis and a mass of 0.45Mgr is applied to the span, producing a theoretical frequency of 5 Hz. From the four octaves logarithmic sine sweep test (frequency range at [1 16] Hz) and the corresponding PPM applied, the natural frequency and the damping
ratio of the specimen are estimated $\dot{f}_n = 4.590 \, Hz$ and $\dot{\zeta}_n = 2.110 \, \%$, respectively. The natural frequency results about 0.5Hz lower than its theoretical counterpart, while the lightly damped nature of the specimen is retained. Using these modal values, the corresponding stiffness and damping of the single DOF structure are found to be $\hat{k}_1 = 1290.90 \, kN/m$ and $\dot{\zeta}_1 = 1.888 \, kN \cdot s/m$, respectively, using Equations (3.45) and (3.46).

Figure 3.25 illustrates the results of the adaptation process, including the identification of the transfer system and the inverse controller. Following the results of the previous test case, again Gaussian white noise excitation of variance $\sigma^2_{x} = 0.05 \, g$ is used as input, filtered with cut-off frequency equal to the Nyquist one ($F_c/2 = 500 \, Hz$), while the same filter (direct-form FIR transposed of 40th order) is applied to the measured shaking table acceleration. As Figure 3.25(a) indicates, the FIR filter of the transfer system has four or five leading terms which can be considered as zeros and this is the delay of the transfer system, while its order (300) is lower than the previous case. Regarding the FIR filter of the inverse controller (again of order 500, Figure 3.25(b)), it is entirely different than the previous case (compare to Figure 3.17(b)). The convolution of the two filters is depicted in Figure 3.25(c), where it is clear that their cascade is an impulse with delay $\Delta = 250$ and amplitude close to (but not equal to) unity. Indeed, this result enforces the need for the integration of an automatic gain controller to the adaptation procedure.

Figures 3.26 – 3.29 show the results of the RTDS tests for the excitations considered. It is noted that in this case, the foundation mass has value $(m_b = 0.3880 \, Mgr)$, in order to maintain the specimen – to – mass ratio equal to 4. Similar comments apply here, as in the previous case. The results are somewhat better (especially in the sinusoidal excitation test) but the amplitude problem remains.
Figure 3.25. (a) Estimated impulse response of the transfer system (on – line). (b) Estimated impulse response of the inverse controller and (c) the convolution of the two impulses (specimen of period $T=0.2s$).
Figure 3.26. Results of the RTDS test for the sinusoidal input. (a) theoretical (black) and achieved (red) absolute acceleration of the foundation and (b) synchronization plot (specimen of period $T = 0.2$ s).

Figure 3.27. Results of the RTDS test for the sinusoidal input. Theoretical (black) and achieved (red) base shear force (specimen of period $T = 0.2$ s).
Figure 3.28. Results of the RTDS test for the Kalamata earthquake (50% scale). (a) theoretical (black) and achieved (red) absolute acceleration of the foundation and (b) synchronization plot (specimen of period $T = 0.2$ s).

Figure 3.29. Results of the RTDS test for the Kalamata earthquake (50% scale). Theoretical (black) and achieved (red) base shear force (specimen of period $T = 0.2$ s).
3.5 A FILTERED – X ADAPTIVE INVERSE CONTROL FRAMEWORK FOR SHAKING TABLES

As already mentioned, the realization of a proper tracking controller is essential in RTDS in order to ensure, with the minimum possible uncertainty, that the transfer system performs as a pure delay. Then, the success of RTDS is dependent to the selected numerical integration algorithm, the applied delay compensation scheme and the type of sensor(s) used for feedback.

In this Section an alternative tracking controller for use in RTDS is introduced, that is based on a modified filtered – X algorithm (Widrow and Wallach 2007). The conventional filtered – X algorithm is a celebrated least mean squares (LMS) algorithm, which has been used in a wide variety of signal processing and vibration control applications. In this work, the original algorithm is modified accordingly by taking into account the target application of interest, while the resulted tracking adaptive inverse controller is assessed by a series of tests in the NTUA LEE shaking table, using two different specimens and acceleration command.

3.5.1 Description of the controller

The adaptive architecture that was proposed in Sections 3.3.1 – 3.3.2 is effective as long as the weights of the forward model and the adaptive inverse controller are adequate and there are no truncation effects. If any of these two conditions is violated, the both FIR filters result biased, leading to a controller which does not minimize the overall system error. Clearly, a possible solution is to use a scheme where the adaptation of the inverse controller would be less sensitive to any form of error in the adaptive model of the transfer system.

This can be accomplished by introducing the design depicted in Figure 3.30, which illustrates the applied online adaptive inverse control framework. As it is seen, adaptation of the inverse controller $iT(z)$ is taking place by minimizing the overall system error, defined as the difference between the achieved response of the shaking table to a delayed reference signal. It follows (Widrow and Wallach 2007) that this system is highly insensitive to errors in the estimated model of the transfer system, $T(z)$: in fact it is not necessary to wait for full convergence of $T(z)$, before starting adaptation of $iT(z)$, as long as the adaptive process is stable.
Figure 3.30. Structure of the applied adaptive inverse controller. Notation: \( T(z) \) is the adaptive model of the transfer system; \( iT(z) \) is the adaptive inverse model of the transfer system; \( k \) is the selected delay for the adaptation. \( \text{command} \) is the reference signal (acceleration); \( \text{achieved} \) is the actual acceleration of the shaking table; \( \text{dither} \) is an external signal used for the adaptation of \( T(z) \). The blocks with dashed arrow lines indicate copies of the corresponding FIR models.

The adaptive inverse controller of Figure 3.30 consists of two phases. In the former (upper part of the figure) the adaptation of \( T(z) \) is taking place by introducing an external signal (dither), while in the latter (lower part of the figure) the adaptation of the inverse controller is realized, by using a copy of the estimated model of the transfer system. These two phases are described below.
3.5.1.1 *Phase 1: adaptive identification of the transfer system*

Figure 3.31 zooms to the forward adaptation part of the adaptive inverse controlled shown at Figure 3.30. This part is a modified version of the Dither C scheme of Widrow and Wallach (Widrow and Wallach 2007). The envisage process uses both the command signal (not shown in Figure 3.31) and an external signal, and adapts the weights of $T(z)$, in order to match its output to the achieved signal of the shaking table. The adaptation algorithm used here is the decorrelated LMS one, which is described in Section 3.3.1 and depicted in Table 3.4.

The meaning of introducing an external signal for adapting the transfer system is that in many cases the command signal may be non–stationary, or limited to a very narrow frequency band (usually the one at which earthquake spectral information is contained). Thus, such a band limited command signal is characterized by correlation properties that may distort the matching of $T(z)$, resulting in poor adaptation.

As seen at Figure 3.31, the transfer system is driven by the combined command and dither signals, while adaptation of $T(z)$ is carried out using the latter. A copy of $T(z)$ is excited by the command signal only and its output is subtracted from the achieved signal of the shaking table and used to calculate the weights of $T(z)$ during the next iteration. Notice that in this way the
adaptation of $T(z)$ does not contain any dynamic components originating from the command signal. It is emphasized that the inclusion of the copied $T(z)$ does not affect the Wiener solution, while at the same time reduces the minimum mean square error.

Several issues arise from the implementation of this scheme to the adaptive modeling of the transfer system, including the selection of the dither signal (input power and spectral components), the stability, the misadjustment and the convergence. An analysis of the process for the case of LMS filter is given in (Widrow and Wallach 2007). However, the use of the decorrelated LMS filter alters the performance of the adaptation significantly and requires further investigation, which is currently ongoing. Here, it is stated that the process of Figure 3.31 with the use of the decorrelated LMS filter accelerates the convergence rapidly.

### 3.5.1.2 Phase 2: adaptive identification of the inverse controller

Having adapted $T(z)$ to the transfer system, the algorithm switches to the adaptation of the inverse controller, $iT(z)$. It is reminded that adaptation takes place in real-time and that it is not necessary to wait for full convergence of $T(z)$, in order to initiate the training of $iT(z)$ (however, with the usage of the decorrelated LMS the convergence of $T(z)$ is not an issue).

The inverse training part of the adaptive controller is illustrated in Figure 3.32. Again a copy of $T(z)$ is utilized, which is excited by the command signal. The output of the copied $T(z)$ is driven to the adaptation process, which, in this case, attempts to minimize the overall system error (the difference between the achieved shaking table signal and the delayed command signal). This algorithm qualifies over the one proposed in Section 3.3.2, in that it is insensitive to errors in $T(z)$ and it can be easily (and safely) implemented in real-time.

The adaptation process in this phase is carried out using the discrete cosine transform (DCT) LMS filter (Diniz, 2008). DCT – LMS belongs to a family of transformation – domain adaptive algorithms that are designed for reducing the effects of large eigenvalue spreads on the correlation matrix of the command signal, speeding up significantly the adaptation time, without reducing adaptation quality.
The DCT – LMS algorithm is presented in Figure 3.33 and Table 3.5. At each step, the tap – delayed input values are transformed by the DCT and the resulted values are normalized by the square root of their power. Accordingly, the equal – power values are convoluted with the LMS weights, which they are adjusted as usually. It is noted that the orthogonalization step is data independent (only filter – order dependent) and, in fact, the DCT matrix can be calculated and stored to memory before the initialization of the adaptation process. The DCT (unitary) transform is given by (see also Table 3.5)

\[ c_n(i,l) = \sqrt{\frac{2}{n}} K_i \cos \left( \frac{i(l+1/2)\pi}{n} \right), \quad i,l = 0,1, \ldots, n-1 \]  

(3.61)

where \( n \) is the order of the filter and \( K_i = 1/\sqrt{2} \) for \( i = 0 \) and unity otherwise.
Table 3.5. Steps of the DCT – LMS algorithm. Parameter $\rho$ is a trimming factor.

<table>
<thead>
<tr>
<th>Step</th>
<th>Description</th>
<th>Equation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Apply DFT (Equation (3.61))</td>
<td>$x_i[t] = \sum_{l=0}^{n-1} c_n(i,l)u[t-l], \ i = 0,1, ..., n-1$</td>
</tr>
<tr>
<td>2</td>
<td>Normalize power$^1$</td>
<td>$v_i[t] = \frac{x_i[t]}{\sqrt{P_i[t] + \varepsilon}}, \ i = 0,1, ..., n-1$</td>
</tr>
<tr>
<td>3</td>
<td>Update $P_i[t]^2$</td>
<td>$P_i[t] = \gamma P_i[t-1] + (1 - \gamma)x_i^2[t]$</td>
</tr>
<tr>
<td>4</td>
<td>Update filter’s weights$^3,4$</td>
<td>$\hat{h}_i[t+1] = \hat{h}_i[t] + \mu e_i[t]v_i[t]$</td>
</tr>
</tbody>
</table>

$^1\varepsilon$ is a small constant
$^2\gamma \in (0,1]$, is the power update factor
$^3\mu$ is the step size
$^4e[t]$ refers to the overall system error

### 3.5.2 Performance assessment

The adaptive inverse control algorithm presented in the previous section is now validated and assessed via actual tests on two distinct specimens. The first corresponds to the structure of Section 3.4.2 ($T = 0.5$s) and the second to a 7Mgr steel frame structure. All tests were conducted in real – time using a sampling period $T_s = 1ms$ and the build in controller of the shaking table (a conventional fixed – gain three variable controller) was set at acceleration mode. Thus, both the excitation signals (command input and dither) and the feedback from the table had acceleration units ($g$).

During tests, emphasis was given to the adaptation of the inverse controller, rather to the one of the transfer system, as this task is already covered in detail in Section 3.4. It is noted however that a lot of effort was paid to the specification of the signals that drive the shaking table, as well as to the processing of the feedback. More specifically, a series of tests was performed using low band filtered white noise at $F_c = 50Hz$ cut – off frequency and it was found that the DCT – LMS algorithm was seriously distorted. This inefficiency in the resulted weights of the inverse controller was observed even in the case where the cut – off frequency was increased up to the
Nyquist one (half the sampling rate). This filtering process took place by utilizing specialized analog devices which were acting on both the excitation and the feedback signals.

A possible explanation of this distortion may be given by taking into account the autocorrelation alteration due to filtering: in the pure white noise the autocorrelation is, at least theoretically, zero everywhere except from the zero lag. On the contrary, the introduction of filtering causes the autocorrelation to attain a sinusoidal – based profile of the form (Bendat and Piersol, 2000),

\[ R_{xx}[\tau] = aB \frac{\sin(2\pi B \tau)}{2\pi B \tau} \quad (3.62) \]

where \( B \) is the bandwidth. During these initial tests, it seemed that signals with autocorrelation as the one described by Equation (3.62) were interfered with the DCT – LMS algorithm, causing the adaptive weights to follow a, more or less, similar curve. While this issue is currently under investigation (in both a theoretical framework and an assessment of the applied hardware), following this initial insight, the input command and the dither were chosen to be white noise of variance similar to that of the applied earthquake (Kalamata), leaving the actual mechanical system (shaking table and specimen) to act as a low pass filter.

### 3.5.2.1 Results for the first specimen

In the series of tests with the specimen of Section 3.4.2 the order of \( T(z) \) was kept constant and equal to \( n_T = 150 \). The order of the inverse controller varied between \( n_{IT} = 100 \) and \( n_{IT} = 300 \), using increments of 50 orders, while in each distinct test the delay \( k \) was chosen to be equal to half the order. The step size and the power update factor of the DCT – LMS algorithm, as well as the adaptation time were kept constant (\( \mu = 5 \times 10^{-5} \), \( \gamma = 0.95 \) and \( T_a = 245s \), respectively, expect in the case of \( n_{IT} = 300 \), at which \( \mu = 1 \times 10^{-5} \), \( \gamma = 0.95 \) and \( T_a = 300s \), respectively). Following a small interval after adaptation, the Kalamata earthquake was subsequently applied to the structure at full scale. During every test, the command acceleration and the achieved acceleration of the shaking table were recorded, together with the converged filters. The results of the tests are illustrated in Figures 3.34 – 3.48.
As a general remark, it seems that the adaptive inverse control scheme of Figure 3.30 exhibits satisfying overall performance in all five tests. In specific, the adaptive modeling of the transfer system is very fast, converging in up to 10 to 15 seconds. Accordingly, the initiation of Phase 2 (Section 3.5.1.2), is taking place smoothly, without any evidence of instabilities and the adaptation manages to converge within the specified time. In fact, as shown at the diagrams of the total errors, the convergence is accomplished somewhere between the 100th and the 150th second. This indicates that the selected value of the DCT – LMS algorithm is successful. However, in all five tests the convolution of $T(z)$ and $iT(z)$ has not resulted in a pure unit delay, probably the result of forcing the adaptive inverse controller to adapt in such a large frequency band.

Focusing on each distinct test, it is assessed in terms of the resulted convolution (especially of the frequency response function (FRF), which is calculated and displayed in the $[0.1, 20]Hz$ band) and the produced synchronization plot during application of the Kalamata earthquake. In the first test ($n_{iT} = 100$ and $k = 50$), the FRF of the convolution clearly exceeds the ±1dB zone (Figure 3.36) in the higher frequency area, while the synchronization plot (Figure 3.35) is rather poor, especially in amplitude terms, as there are some clear differences between the reference and the achieved acceleration in several peaks. The second test ($n_{iT} = 150$ and $k = 75$) extracted significantly improved results, both in the FRF, which stays within the prescribed zone throughout the whole frequency band (Figure 3.39), and in the synchronization plot (Figure 3.38). Very similar is also the performance of the third test ($n_{iT} = 200$ and $k = 100$), which shows better synchronization plot (Figure 3.41), compared to the one of the second test (observe the peak matching between the reference and the achieved acceleration during Kalamata earthquake), but slightly worse FRF in the area near 20Hz. In the fourth test ($n_{iT} = 250$ and $k = 125$) the results are rather poor (Figures 3.43 – 3.45), indicating that either estimation noise is significant at this order, or that the tuning of the DCT – LMS is not appropriate. That is why the parameters of the fifth test ($n_{iT} = 300$ and $k = 150$, Figures 3.46 – 3.48) were altered, producing good performance of the adaptive inverse controller, yet by no means better than the one of tests 2 and 3.
Figure 3.34. Reference/achieved shaking table acceleration and total error during adaptation for $n_{tr} = 100$ and $k = 50$ (first specimen). The reference acceleration has been shifted $k$ steps backwards.

Figure 3.35. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{tr} = 100$ and $k = 50$ (first specimen). The reference acceleration has been shifted $k$ steps backwards. Right Figure: synchronization plot of the signals on the left.
Figure 3.36. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 100$ and $k = 50$ (first specimen).

Figure 3.37. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 150$ and $k = 75$ (first specimen). The reference acceleration has been shifted $k$ steps backwards.
Figure 3.38. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{IT} = 150$ and $k = 75$ (first specimen). The reference acceleration has been shifted $k$ steps backwards. Right Figure: synchronization plot of the signals on the left.

Figure 3.39. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 150$ and $k = 75$ (first specimen).
Figure 3.40. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 200$ and $k = 100$ (first specimen). The reference acceleration has been shifted $k$ steps backwards.

Figure 3.41. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{IT} = 200$ and $k = 100$ (first specimen). The reference acceleration has been shifted $k$ steps backwards; Right Figure: synchronization plot of the signals on the left.
Figure 3.42. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 200$ and $k = 100$ (first specimen).

Figure 3.43. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 250$ and $k = 125$ (first specimen). The reference acceleration has been shifted $k$ steps backwards.
Figure 3.44. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{TR} = 250$ and $k = 125$ (first specimen). The reference acceleration has been shifted $k$ steps backwards; Right Figure: synchronization plot of the signals on the left.

Figure 3.45. Impulse response and $\pm 1$ dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{TR} = 250$ and $k = 125$ (first specimen).
Figure 3.46. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 300$ and $k = 150$ (first specimen). The reference acceleration has been shifted $k$ steps backwards.

Figure 3.47. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{IT} = 300$ and $k = 150$ (first specimen). The reference acceleration has been shifted $k$ steps backwards. Right Figure: synchronization plot of the signals on the left.
Figure 3.48. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{iT} = 300$ and $k = 150$ (first specimen).

As both test 2 and test 3 produced very close results the former was chosen as the best out of all mostly because the smaller order and delay. The second series of tests with this specimen was devoted to an effort of decreasing the delay, as the adaptive inverse controller is intended for use in RTDS and smaller delays are more easily compensated. Figures 3.49 – 3.63 display the results of this series of tests, at which the order was kept constant ($n_{iT} = 150$) and the delay was selected as $k = 60, 50, 40, 30$ and 20.

A first observation is that the impulse response of the convolution gradually decreases from about 0.78, when $k = 60$ (Figure 3.51), to about 0.60 when $k = 20$ (Figure 3.63). However, the corresponding FRFs stay within the ±1dB zone, with a few exceptions near 20Hz. Up to $k = 30$ (Figures 3.49 – 3.60) the overall results are close to each other (satisfying synchronization plots and peak performance) and it can be declared that the case $n_{iT} = 150$ and $k = 30$ is a good candidate for performing RTDS tests with this specimen. It is also clear that the algorithm finds the $k = 20$ delay small and starts to degrade its performance (for the same testing conditions).
Figure 3.49. Reference/achieved shaking table acceleration and total error during adaptation for \(n_{IT} = 150\) and \(k = 60\) (first specimen). The reference acceleration has been shifted \(k\) steps backwards.

Figure 3.50. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for \(n_{IT} = 150\) and \(k = 60\) (first specimen). The reference acceleration has been shifted \(k\) steps backwards. Right Figure: synchronization plot of the signals on the left.
Figure 3.51. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 150$ and $k = 60$ (first specimen).

Figure 3.52. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 150$ and $k = 50$ (first specimen). The reference acceleration has been shifted $k$ steps backwards.
Figure 3.53. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{IT} = 150$ and $k = 50$ (first specimen). The reference acceleration has been shifted $k$ steps backwards; Right Figure: synchronization plot of the signals on the left.

Figure 3.54. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 150$ and $k = 50$ (first specimen).
Figure 3.55. Reference/achieved shaking table acceleration and total error during adaptation for \( n_{IT} = 150 \) and \( k = 40 \) (first specimen). The reference acceleration has been shifted \( k \) steps backwards.

Figure 3.56. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for \( n_{IT} = 150 \) and \( k = 40 \) (first specimen). The reference acceleration has been shifted \( k \) steps backwards; Right Figure: synchronization plot of the signals on the left.
Figure 3.57. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{1T} = 150$ and $k = 40$ (first specimen).

Figure 3.58. Reference/achieved shaking table acceleration and total error during adaptation for $n_{1T} = 150$ and $k = 30$ (first specimen). The reference acceleration has been shifted $k$ steps backwards.
Figure 3.59. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{IT} = 150$ and $k = 30$ (first specimen). The reference acceleration has been shifted $k$ steps backwards; Right Figure: synchronization plot of the signals on the left.

Figure 3.60. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 150$ and $k = 30$ (first specimen).

As both test 2 and test 3 produced very close results the former was chosen as the best out of all.
Figure 3.61. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 150$ and $k = 20$ (first specimen). The reference acceleration has been shifted $k$ steps backwards.

Figure 3.62. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{IT} = 150$ and $k = 20$ (first specimen). The reference acceleration has been shifted $k$ steps backwards; Right Figure: synchronization plot of the signals on the left.
Figure 3.63. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 150$ and $k = 20$ (first specimen).

Figure 3.64. Views of the second specimen used for the tests.
### 3.5.2.2 Results for the second specimen

Figure 3.64 displays the steel frame structure that used for a further assessment of the adaptive inverse controller. The frame consists of four fixed-ended columns that are interconnected by a rigid steel diaphragm. The total weight of the specimen is 7Mgr. Bolts are used to assemble the members of the specimen. For determining the dynamic characteristics of the specimen a logarithmic sine sweep excitation was applied. The acceleration amplitude of the sine sweep was 0.05g and the frequency range was 1 to 32Hz with a rate of 1 octave per minute. The frequency of the specimen was found 7.46Hz and the corresponding damping ratio was $\zeta=4\%$. Compared to the first specimen, this one is stiffer with higher damping.

The same procedure as before was followed here, regarding the validation of the adaptive inverse controller and the results are displayed in Figures 3.65 – 3.79. Again, the adaptive inverse control scheme of Figure 3.30 exhibits satisfying overall performance in all five tests, converging in up to 10 to 15 seconds. The initiation of Phase 2 is also taking place smoothly, without any evidence of instabilities. As before, in all five tests the convolution of $T(z)$ and $iT(z)$ has not resulted in a pure unit delay, for the same reasons. The best performance, in terms of synchronization plots and FRF was adopted for the third test ($n_{tr} = 200$ and $k = 100$).
Figure 3.65. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 100$ and $k = 50$ (second specimen). The reference acceleration has been shifted $k$ steps backwards.

Figure 3.66. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{IT} = 100$ and $k = 50$ (second specimen). The reference acceleration has been shifted $k$ steps backwards; Right Figure: synchronization plot of the signals on the left.
Figure 3.67. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 100$ and $k = 50$ (second specimen).

Figure 3.68. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 150$ and $k = 75$ (second specimen). The reference acceleration has been shifted $k$ steps backwards.
Figure 3.69. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for \( n_{IT} = 150 \) and \( k = 75 \) (second specimen). The reference acceleration has been shifted \( k \) steps backwards; Right Figure: synchronization plot of the signals on the left.

Figure 3.70. Impulse response and ±1dB zone frequency response of the convolution between the adapted \( T(z) \) and \( iT(z) \) for \( n_{IT} = 150 \) and \( k = 75 \) (second specimen).
Figure 3.71. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 200$ and $k = 100$ (second specimen). The reference acceleration has been shifted $k$ steps backwards.

Figure 3.72. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{IT} = 200$ and $k = 100$ (second specimen). The reference acceleration has been shifted $k$ steps backwards; Right Figure: synchronization plot of the signals on the left.
Figure 3.73. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 200$ and $k = 100$ (second specimen).

Figure 3.74. Reference/achieved shaking table acceleration and total error during adaptation for $n_{IT} = 250$ and $k = 125$ (second specimen). The reference acceleration has been shifted $k$ steps backwards.
Shaking table test techniques and fault rupture box testing for SSI

Figure 3.75. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for $n_{IT} = 250$ and $k = 125$ (second specimen). The reference acceleration has been shifted $k$ steps backwards; Right Figure: synchronization plot of the signals on the left.

Figure 3.76. Impulse response and ±1dB zone frequency response of the convolution between the adapted $T(z)$ and $iT(z)$ for $n_{IT} = 250$ and $k = 125$ (second specimen).
Figure 3.77. Reference/achieved shaking table acceleration and total error during adaptation for \( n_{IT} = 300 \) and \( k = 150 \) (second specimen). The reference acceleration has been shifted \( k \) steps backwards.

Figure 3.78. Left figure: reference/achieved shaking table acceleration during Kalamata earthquake for \( n_{IT} = 300 \) and \( k = 150 \) (second specimen). The reference acceleration has been shifted \( k \) steps backwards; Right Figure: synchronization plot of the signals on the left.
3.6 CONCLUSIONS

This work aimed at developing a novel framework for effective RTDS of the horizontal SSI problem, based on adaptive signal processing and parameter estimation techniques. Towards this goal, the LEE – NTUA research team designed two different processes, shown in Figure 3.8 and Figure 3.30, respectively. The main features of the former are summarized as follows:

1. Since the dynamics of the transfer system must be compensated, including any delay present, an adaptive controller is designed and identified, either off – line, or on – line. Towards this point, an estimate of the transfer system’s impulse response is required. This is a crucial step of the RTDS process and it is considered successful when the cascade of the two estimated FIR models results to a delayed version of the unit impulse response (Kronecker’s delta function). Notice that this procedure may increase the delay; however this is unavoidable when the transfer system is non – minimum phase (as it usually is).

2. To compensate the effects of the delay, a corresponding predictor is placed prior to the numerical substructure. This predictor handles the part of the relative acceleration of the specimen that cannot be a priori known.
The process is designed to work in acceleration mode: not only acceleration commands are given to the transfer system, but acceleration feedback is also utilized, dropping the need for load cells.

While the results of the first scheme promising, the corresponding adaptation process was prone to errors and other truncation effects caused by the inappropriate tuning of the forward and inverse filters. To this, a second controller was designed, on the basis of a modified filtered – X algorithm, which utilizes the decorrelated LMS method for the forward tuning and the DCT – LMS for the inverse tuning. The proposed framework is implemented in three faces:

**Phase I**: adaptive identification of the plant dynamics (transfer system plus structure) → impulse response of the system that relates the command acceleration to the achieved acceleration of the shaking table.

**Phase II**: adaptive identification of the inverse plant dynamics → estimation of the inverse controller.

**Phase III**: implementation of the adaptive inverse controller.

The second adaptive inverse controller tends to minimize the overall system error. Thus, even if the plant estimate is not a perfect match of the true one, the converged inverse controller is relatively insensitive to this error. The presented results suggest further effort towards the tuning of this controller and especially the DCT – LMS algorithm (likely also the associated hardware), in order to handle low – pass input commands, the use of which will focus adaptation to the frequency band of applications related to earthquake engineering.
4 APPLICATION OF FHT TECHNIQUE FOR SSI PROBLEMS AT UNIVBRIS

4.1 DEVELOPMENT OF THE REAL-TIME SUBSTRUCTURE TEST METHOD FOR THE SHAKING TABLE

In Chapter 4, the real time dynamic substructure test method is extended for shaking table studies using the shaking table of the Bristol Laboratory for Advanced Dynamics Engineering (BLADE) at the University of Bristol as the experimental transfer system. This apparatus consists of a 6 tonne, 3m by 3m cast-aluminium seismic platform with 21 tonne payload. Mounted within a 100 tonne concrete block that is itself secured to bedrock, the platform is driven by eight 70kN servo-hydraulic actuators of 0.3m stroke giving full control of motion in all six degrees of freedom simultaneously. Substructuring occurs in a single horizontal direction, all other rotational and translational degrees of freedom are controlled to zero.

4.1.1 Model of the shaking table system

Transfer systems are used to impose force or displacement on a test specimen and to imitate the constraint dynamics between the numerical model and physical model. To achieve near-perfect synchronization of the interface response between the physical specimen and the numerical model, a good controller is needed to compensate for the dynamics of the transfer systems. Transfer systems are typically single actuators (electric or hydraulic) and include their proprietary controller such as PID controller. With the exception of a phase lag, an actuator normally has a good performance at low frequency range (e.g., 0-10 Hz). To improve the phase response a number of delay compensation algorithms have been proposed. Compared with a single actuator a shaking table has more complex dynamic characteristics. The large mass of the
shaking table limits the operation frequency bandwidth. Indeed, both a phase lag and a magnitude error exist in the frequency range of interest. Delay compensation cannot adequately compensate for such dynamics. Thus, there is a requirement for the development of a controller for shaking table substructuring that is capable of dealing with these more complex dynamics.

To understand and control complex systems, some prior knowledge about the plant is required. Mathematical models of physical system components are key to the analysis and design of a controller. The dynamic behaviour is generally described by ordinary differential equations. The nonlinearities of the hydraulic system are linearised in this chapter, which allow us to use Laplace transform methods in system analysis. Figure 4.1 shows the main components of shaking table testing system. These include electrohydraulic servovalves, a servo-hydraulic actuator, table and its fixtures and fittings, and a specimen. The quantitative mathematic model for each subcomponent will be presented below.

Figure 4.1. A schematic of shaking table testing system.
Electro-hydraulic servovalves

Electro-hydraulic servovalves are widely used in hydraulic actuation. The three-stage servovalve with the type number G761-3015 manufactured by Moog, Inc is used on the University of Bristol 6-axis shaking table. The product literature (Thayler (1958), (1959)) from Moog, Inc indicates that the three-stage servovalve can be described by a second-order system.

\[
G_v(s) = \frac{k_v \omega_v^2}{s^2 + 2\xi_v \omega_v s + \omega_v^2} \quad (4.1)
\]

where \(x_v\) is the valve (spool) displacement from the neutral position \((-1 \leq x_v \leq 1\)), \(u\) is the electrical command signal to the servovalve, \(k_v\) is the valve gain, \(\xi_v\) is the damping ratio, \(\omega_v\) is the frequency of the second-order system, and \(s\) is Laplace-transform variable. For low frequencies, the servovalve dynamics can be approximated by a constant:

\[
G_v(s) = k_v \quad (4.2)
\]

Although the transfer function of servovalve has a nearly flat magnitude in a frequency band (0-30Hz for the servovalve used in this work), a phase lag (also known as a time delay) exists, which influences the stability. To describe the dynamics of servovalve at low frequency more accurately, a first-order model is typically used. The servovalve transfer function is given by:

\[
G_v(s) = \frac{k_v \omega_v}{s + \omega_v} \quad (4.3)
\]

where \(\omega_v = 1/\tau_v\), \(\tau_v\) is the servovalve time constant.

The flow characteristics of the servovalve (i.e., the relationship between spool displacement, controlled flow, and pressure drop across the load) are given by the following general flow equation (Merrit (1967)):

\[
Q_L = C_d w x_v \sqrt{\frac{1}{\rho} \left( \frac{p_s - x_v}{|x_v|} p_L \right)} \quad (4.4)
\]
where \( p_L \) is the pressure drop across the load, \( Q_L \) is the controlled flow through the load, \( x_v \) is the valve (spool) displacement from the neutral position, \( p_s \) is the system supply pressure, \( C_d \) is the coefficient of discharge orifices, \( w \) is the opening or area gradient of the valve orifices (rate of change of orifice area with spool displacement) and \( \rho \) is the fluid density. The nonlinear flow equation can be linearised with respect to an operating point producing the following flow equation:

\[
Q_L = K'_q x_v - K'_p p_L
\]  

(4.5)

where \( K'_q \) is the valve flow gain and \( K'_p \) is the valve flow-pressure gain. The operating point that is typically used to evaluate these valve coefficients is the origin (i.e. \( Q_L = p_L = x_v = 0 \)).

4.1.1.2 Hydraulic actuator

Combinations of servovalves and hydraulic actuators are common hydraulic power elements in large capacity test equipment. The fundamental equations that govern the behaviour of a hydraulic actuator are given by the following relationship:

\[
Q_L = A_p x_v + C_l p_L + \frac{V}{4\beta_e} s p_L
\]  

(4.6)

where \( A_p \) is the area of the piston, \( C_l \) is the total leakage coefficient of the piston, \( V \) is the total volume of fluid under compression in both actuator chambers, and \( \beta_e \) is the effective bulk modulus of the system (including oil, entrapped air, mechanical compliance of the champers), and \( x_v \) is displacement of actuator.

Table-specimen interaction (TSI) is another potential problem in a shaking table testing system. Difficulties associated with TSI can be circumnavigated by minimising the specimen mass with respect to the table mass. In this specific tests of this work, the maximal specimen mass is less than 60kg and the table mass is bigger than 6000kg. Hence, TSI is assumed to be negligible. Using Newton’s second law to the forces on the piston, the resulting force equation of the table together with the piston is:
\[ A_p p_L f_c = m_t s^2 x \]  
(4.7)

where \( m_t \) is the total mass of table, piston and its attachments, and \( f_c \) is friction force on the piston and table. Normally this force is neglected in the analysis of shaking table systems.

### 4.1.1.3 Proprietary controller

The popular Proportional-Integral-Derivative (PID) controller widely used in hydraulic servovalve systems to provide a practical controller structure with three adjustable parameters. The electrical command signal \( u(s) \) from such a controller is expressed as:

\[ u(s) = (k_p + \frac{k_i}{s} + k_d s)e(s) \]  
(4.8)

Here \( k_p, k_i, k_d \) are the proportional, integral, and derivative gains respectively. The error signal \( e(s) \) is defined as the difference between the command and measured displacements:

\[ e = r - y \]  
(4.9)

where \( r \) is expected signals, and \( y \) is achieved signals.

In most practical applications, the integral and derivative gains are set to small magnitudes and the controller is equivalent to a straight proportional controller. Therefore, in the following analysis the PID controller is expressed as:

\[ u(s) = k_p e(s) \]  
(4.10)

Compare with single actuator, shaking table has a lower oil column resonant frequency because of the large table mass. To decrease the influence of the resonant frequency, an advanced additional controller (such as three-variable-controller, accelerator-controller, etc.) is often used in conjunction with the PID controller.
4.1.2 Linear system analysis

Combining the mathematical model of servovalve (Equation 4.1, Equation 4.2), the flow characteristics (Equation 4.4), the actuator (Equation 4.5), the shaking table (Equation 4.6) and the controller (Equation 4.9) produces a model of the entire shaking table testing system. The block diagram of the entire system is shown in Figure 4.2.

![Block diagram of shaking table system control.](image)

Figure 4.2. Block diagram of shaking table system control.

In this way, the open loop transfer function \( G_e \) from the error signal \( e(s) \) to the achieved displacement \( y(s) \) can be obtained. Using Equation 4.2 for constant servovalve dynamics, the open loop transfer function is:

\[
G_e = \frac{y(s)}{e(s)} = \frac{k_q \omega_0^2}{s^2 + 2 \xi_0 \omega_0 s + \omega_0^2} \tag{4.11}
\]

Using Equation 4.3 for first-order servovalve dynamics, the open loop transfer function is:

\[
G_e(s) = \frac{k_q \omega_0^2 \omega_v}{(s + \omega_v)(s^2 + 2 \xi_0 \omega_0 s + \omega_0^2)} \tag{4.12}
\]

Using Equation 4.4 for second-order servovalve dynamics, the open loop transfer function is:

\[
G_e = \frac{k_q \omega_0^2 \omega_v^2}{(s^2 + 2 \xi_0 \omega_0 s + \omega_0^2)} \left[ \frac{\beta_c}{\sqrt{V_m}} \right]^2 \tag{4.13}
\]

\[
\begin{align*}
\omega_0 &= 2 A_p \sqrt{\frac{\beta_c}{V m_i}} \\
\xi_0 &= \frac{k_c}{A_p \sqrt{V_t}} 
\end{align*} \tag{4.14}
\]
where $\omega_0$ and $\xi_0$ are the oil column resonant frequency and natural damping ratio of shaking table, $k_q = k'_q k_i / A_p$, and $K_c = K'_c + C_i$ is the total flow pressure coefficient. Taking into account the controller in Equation 4.9, the closed loop transfer function $G_{ST}$ from the expected signal ($r$) to the achieved displacement ($y$) is obtained. For a constant, first order, and second order servovalve the transfer functions are, respectively:

$$G_{ST}(s) = \frac{y(s)}{r(s)} = \frac{k_q k_p \omega_0^2}{s^2 + 2 \xi_0 \omega_0 s + \omega_0^2 \omega_v}$$ (4.15)

$$G_{ST}(s) = \frac{k_q k_p \omega_0^2 \omega_v}{(s + \omega_v)(s^2 + 2 \xi_0 \omega_0 s + \omega_0^2 \omega_v)} + k_q k_p \omega_0^2 \omega_v$$ (4.16)

$$G_{ST}(s) = \frac{k_q k_p \omega_0^2 \omega_v}{(s^2 + 2 \xi_0 \omega_0 s + \omega_0^2 \omega_v)(s^2 + 2 \xi_0 \omega_0 s + \omega_0^2 \omega_v)} + k_q k_p \omega_0^2 \omega_v$$ (4.17)

The third-order system in Equation 4.15 can be resolved into a second-order system and a first-order system:

$$G_{ST}(s) = \frac{\omega_1^2 \omega_2}{(s^2 + 2 \xi_1 \omega_1 s + \omega_1^2 \omega_1)(s + \omega_2)}$$ (4.18)

$$\begin{align*}
\omega_1^2 \omega_2 &= k_q k_p \omega_0^2 \\
2 \xi_1 \omega_1 \omega_2 + \omega_1^2 &= \omega_0^2 \\
2 \xi_0 \omega_1 + \omega_2 &= 2 \xi_0 \omega_0
\end{align*}$$ (4.19)

Equation 4.18 shows that the behaviour of shaking table without the consideration of the servovalve dynamics includes a first order term. Therefore, the fourth-order system in Equation 4.16 can be resolved into two first-order systems and a second-order system:

$$G_{ST}(s) = \frac{\omega_1^2 \omega_3 \omega_4}{(s^2 + 2 \xi_3 \omega_3 s + \omega_3^2 \omega_3)(s + \omega_4)(s + \omega_4)}$$ (4.20)
\[
\begin{align*}
\omega_3^2 \omega_4 \omega_5 &= k_q k_p \omega_v^2 \\
2 \xi_3 \omega_3 \omega_4 \omega_5 + \omega_3^2 (\omega_4 + \omega_5) &= \omega_0^2 \omega_v \\
\omega_4 \omega_5 + 2 \xi_4 (\omega_4 + \omega_5) + \omega_5^2 &= 2 \xi_0 \omega_v \omega_0 + \omega_v^2 \\
2 \xi_5 \omega_0 + \omega_4 + \omega_5 &= 2 \xi_0 \omega_0 + \omega_v
\end{align*}
\] (4.21)

The fifth-order system in Equation 4.17 can be resolved into two second-order systems and a first-order system:

\[
G_{ST}(s) = \frac{\omega_b^2 \omega_0^2 \omega_v}{(s^2 + 2 \xi_0 \omega_b s + \omega_b^2)(s^2 + 2 \xi_0 \omega_v s + \omega_v^2)(s + \omega_b)}
\] (4.22)

\[
\begin{align*}
\omega_0^2 \omega_2 \omega_3 &= k_q k_p \omega_v^2 \\
\omega_0^2 \omega_4 + (2 \xi_0 \omega_0 \omega_2 + 2 \xi_0 \omega_0 \omega_4) \omega_0 &= \omega_0^2 \omega_v \\
2 \xi_0 \omega_0 \omega_2 + 2 \xi_0 \omega_0 \omega_4 + (\omega_2^2 + 2 \xi_0 \omega_0 \omega_2 + \omega_4^2) \omega_0 &= 2 \xi_0 \omega_0 \omega_0 + 2 \xi_0 \omega_0 \omega_v \\
\omega_0^2 + 4 \xi_0 \omega_0 \omega_2 + \omega_2^2 + 2 \xi_0 \omega_0 \omega_4 + \omega_4^2 &= \omega_0^2 + 4 \xi_0 \omega_0 \omega_0 + \omega_v^2 \\
2 \xi_0 \omega_0 + 2 \xi_0 \omega_4 + \omega_4 &= 2 \xi_0 \omega_0 + 2 \xi_0 \omega_0
\end{align*}
\] (4.23)

Equation 4.18, Equation 4.20 and Equation 4.22 show that the dynamics of shaking table system are a combination of several second-order and first-order systems. It is the frequencies and damping ratios of these subsystems that determine the behaviour of shaking table. The lower the damping ratios (\(\xi_1\) or \(\xi_3, \xi_4\)) and cut-off frequencies (\(\omega_1, \omega_2, \omega_4\)) (i.e. where the phase lag equals90\(^\circ\)) the poorer the accuracy of system. When the frequencies and damping ratios are too low, the substructure system will be unstable.

### 4.1.3 Shaking table system identification

The values of all the parameters in Equation 4.15, Equation 4.16 and Equation 4.17 can be theoretically determined using the technical literature from the manufacturer, and using experimental data (see Williams et al (2001) and Zhao et al (2006)). However, the workings of the proprietary shaking table controller in use at the University of Bristol are obscure as they are commercially sensitive. Consequently the controller is difficult to model accurately. Additionally, the hydraulic system is strongly nonlinear. Thus, the near-linear model is valid.
only in a limited frequency band. Also, Equation 4.18, Equation 4.20 and Equation 4.22 show that the linear shaking table model can be described mathematically only if the polynomial coefficients of the transfer function denominator are available.

A sine sweep from 0.1 to 20Hz over 100s was used to estimate experimentally a third-order and a fourth-order transfer function for the shaking table. Equation 4.15, Equation 4.16 and Equation 4.17 show that the constant term of denominator of the transfer function equals that of the numerator. However, the real system is a more complex, consisting of a proprietary controller and associated hardware etc. To optimise the model for this system, a slight adjustment of the numerator of the estimated transfer function was made. The resulting transfer functions for the shaking table are:

\[
3^{\text{rd}} \text{order: } G_{ST}(s) = \frac{1.8e5}{s^3 + 88.36s^2 + 5375s + 1.745e5} \tag{4.24}
\]

\[
4^{\text{th}} \text{order: } G_{ST}(s) = \frac{7.53e7}{s^4 + 406.90s^3 + 3.75e4s^2 + 2.209e6s + 7.30e7} \tag{4.25}
\]

A comparison of the frequency response of the estimated model and experimental data is shown in Figure 4.3 and Figure 4.4. The third-order transfer function provides a good model up to about 7Hz. After this frequency, the magnitude and the phase lag between the model and experiment diverge. The fourth-order transfer function provides an excellent model up to about 12Hz and an adequate model up to 20Hz. This includes the typically frequency range of interest for shaking table substructuring. The time history response in Figure 4.5 and Figure 4.6 also show that the third-order and fourth-order transfer function closely match the response of the real system over the frequency range of the test (0-10Hz in this study). The detail view of Figure 4.5 and Figure 4.6 illustrate that fourth-order transfer function provides a better model across a wider frequency range than third order.
Figure 4.3. Third-order transfer function of the shaking table.

Figure 4.4. Fourth-order transfer function of the shaking table.

Figure 4.5. Shaking table time history simulation using the third order model.
4.1.4 Substructure testing system

A complete model of an entire substructure testing system can be developed using the dynamic model of the shaking table that was presented in the preceding sections. This fundamental system model can then be used to analyse the stability of the testing system, assess the performance of shaking table controllers. The system model is described below and, moreover, used to develop a new and improved control methodology called Full State Control via Simulation (FSCS) in following chapters.

4.1.4.1 Modelling of a substructure test

A generalised multi degree-of-freedom (DOF) shaking table substructuring system is shown fully assembled in Figure 4.7(a) and decomposed into its physical and numerical components in Figure 4.7(b). The numerical model is compiled on to a real-time processing board (herein, a Dspace DS1103 board is used) using a numerical computation program (Simulink). The numerical substructure is acted upon by both an excitation load \( (r) \) and a reaction force generated by the physical component \( (f_{\text{reaction}}) \). A numerical displacement at the interface between numerical and physical is calculated under these loads. The interface displacement is used to derive the shaking table command signal \( (u_o) \) which is subsequently imposed using an outer loop controller. The experimental rig includes the physical parts and the shaking table. The
load cell transducer is the interface between the numerical and physical parts, measuring the reaction force resulting from physical part which is fed back to the numerical part. The shaking table applies the physical displacement of interface \((y_{IP})\) on the physical parts, which, for accuracy, must be synchronized with the numerical displacement \((y_{IN})\) via the outer loop controller.

![Diagram of the substructure testing using shaking table](image)

**Figure 4.7. Principle of the substructure testing using shaking table.**

The linear relationships of different parts of this substructured system are synthesized as follows. The numerical response of interface \((y_{IN})\), the command signal \((u_o)\), the physical response of interface \((y_{IP})\) and the reaction forces from the physical parts are given by:

\[
y_{IN} = y_r + y_f = G_{No} r + G_{Nf} f_{reaction}
\]  
(4.26)

\[
u_o = G_{OC} y_{IN}
\]  
(4.27)

\[
y_{IP} = G_{Sp} u_o
\]  
(4.28)

\[
f_{reaction} = G_P y_{IP}
\]  
(4.29)
Following on from Equation 4.26 to Equation 4.29, the control loop of the substructured system is illustrated in Figure 4.8.

![Block diagram](https://via.placeholder.com/150)

**Figure 4.8. Block diagram of a typical control strategy for a substructured system.**

Hence, Figure 4.8 gives the closed-loop transfer function:

\[
\frac{y_{IP}}{r} = \frac{G_{OC}G_{ST}G_{Nr}}{1 + G_{OC}G_{ST}G_{Ny}G_{P}}
\]  

(4.30)

In an ideal substructuring implementation, the numerical and physical substructures will interact seamlessly to replicate the emulated system exactly. That means \(G_{OC}G_{ST} = 1\). Then the exact transfer function between the physical response of interface and reference signal is:

\[
\frac{y_{IP}}{r} = \frac{G_{Nr}}{1 + G_{Ny}G_{P}}
\]  

(4.31)

In reality, however, a useful distinction which recognizes the imperfect control of the transfer system can be made between the ‘desired’ numerical displacement and the ‘achieved’ displacement applied by the shaking table.

### 4.1.4.2 Analysis of a substructuring test system

An estimate of the feasibility of a substructuring test can be obtained following a consideration of its stability. From Figure 4.8, the closed-loop transfer function of a substructured system without outer loop controller is given:
\[ y_{IP} = \frac{G_S G_{Nf}}{1 + G_S G_{Nf} G_P} \]

and the corresponding closed-loop characteristic equation (CLCE) is:

\[ 1 + G_S G_{Nf} G_P = 0 \]

As known from Nyquist stability criterion, the phase margin of \( G_S G_{Nf} G_P \) determines the stability of a control system. Hence, by considering the phase margin generated by only \( G_{Nf} G_P \), the performance of the combined controller and the shaking table that is necessary for stability can be assessed. Figure 4.9 shows the Bode plot of \( G_{Nf} G_P \) of a typical substructured system. The phase margin \( \phi_{pm} \) is measured at frequency \( \omega_c \) where the loop gain is unity. When the phase lag \( \phi_c \) of the shaking table (shown in Figure 4.10) at frequency \( \omega_c \) exceeds \( \phi_{pm} \), the substructured system has a negative phase margin which guarantees instability. It is worth noting that phase lag not only diminishes stability but also decreases accuracy particularly for the lightly damped systems apparent in civil engineering.

![Bode Diagram](image)

**Figure 4.9.** Bode plot of \( G_{Nf} G_P \).
In actuator-based substructuring tests (i.e. without a shaking table), actuator phase lag can be compensated for successfully by using a delay compensation algorithm (Horiuchi et al (1999), Darby et al (2002), Wallace et al (2005a)). However, it has not been established whether such algorithms are of any use when a shaking table is employed as the transfer system. This is explored in the following section.

### 4.1.5 Preliminary substructuring tests

To investigate the effectiveness of delay compensation for shaking table substructure tests, the two DOF systems shown in Figure 4.11 is taken as an example. For practical reasons, and since the tests are intended to focus on the effectiveness of the controller, a simplification is made whereby both the ‘physical’ and numerical parts of this system are modelled numerically. Only the shaking table transfer system exists in the physical world. Note, however, that in the following discussion the terminology and the subscripts $m$ and $p$ are retained in order to distinguish between the different components of the system.
4.1.5.1 Test configuration

The experiment is conducted using the system shown in Figure 4.12. The numerical part, physical part and delay compensation algorithm are all modelled numerically and generate the control signal (u) for the shaking table proprietary controller. The numerical simulation is compiled onto an outer loop dSpace DS1103 real-time control board and run at time steps of 1 millisecond. A sensor is used to measure the acceleration of the shaking table (aIP) which is fed back to the numerical simulation via the analogue to digital converters (ADC) of the dSpace I/O breakout box. The reaction force from the physical DOF is derived from the measured acceleration. The linear transfer function of different parts in Figure 4.12 is synthesized as follows:

\[
G_{Nr} = \frac{c_Ns + k_N}{m_Ns^2 + c_Ns + k_N} \tag{4.34}
\]

\[
G_{Nf} = \frac{1}{m_Ns^2 + c_Ns + k_N} \tag{4.35}
\]

\[
G_p = \frac{m_p(c_ps + k_p)}{m_ps^2 + c_ps + k_p} \tag{4.36}
\]

where \(m_N, c_N, k_N\) are the mass, damping and stiffness of the numerical part, and \(m_p, c_p, k_p\) are the mass, damping and stiffness of the physical part. A constant delay compensation algorithm is adopted in this system (Horiuchi et al (1999)), which is described by:
where \( a_0 = 4, a_1 = -6, a_2 = 4, a_3 = -1 \), \( u \) is the control signal, \( y_{IN0} \) is the present calculated displacement of the interface, \( y_{INj} \) are calculated displacements of the interface at previous time increments \( j\tau \), and \( \tau \) is the time delay of the shaking table system. A reasonable estimate for the constant time delay of the University of Bristol can be obtained from Figure 4.4 as 28.5 milliseconds.

\[
 u = \sum_{j=0}^{3} a_j y_{INj} \quad (4.37)
\]

Figure 4.12. Preliminary substructuring test set-up.

### 4.1.5.2 Preliminary results

Two systems were tested in this section: System 1 and System 2. The parameters of System 1 are \( m_p = 56.38 \text{kg}, \ m_N = 62.64 \text{kg}, \ \omega_N = \omega_p = 1.936 \text{Hz}, \ \xi_N = \xi_p = 0.05 \). The mass and frequencies of System 2 are same as System 1, but with \( \xi_N = \xi_p = 0.25 \). The displacement record of the El Centro earthquake (NS component of the 1940 El Centro, Station: 117 El Centro Array #9, Component 270°) was chosen to be the reference excitation signal.

System 1 is a lightly damped system with 5% damping ratio, which has a small phase margin, and hence poses a challenge for substructured system control because of the phase lag of transfer system. Figure 4.13 displays the numerical displacement response of interface (called the ‘desired displacement’), the displacement response of shaking table (called the ‘achieved displacement’) and the command signals generated from delay compensation controller. Note that the desired and achieved displacement lines lie on top of each other. Delay compensation clearly improves the performance of shaking table and allows the interface displacement to be actuated...
Shaking table test techniques and fault rupture box testing for SSI

successfully. Figure 4.14 and Figure 4.15 compare the substructured displacement and substructured reaction force of physical DOF to the corresponding ‘emulated’ data obtained via simulation of the whole system numerically (i.e. without the shaking table transfer system). As can be seen, excellent agreement of both the displacement and the reaction force are achieved between the test results and the emulated solution.

Figure 4.13. Shaking table displacement for System 1.

Figure 4.14. Displacement of physical part of System 1.
Compared with System 1, System 2 is a relatively high damped system with 25% damping ratio. As seen in Figure 4.16, at 3.2 seconds into the test, instability occurs causing the shaking table command signal to grow without bound. The safety mechanisms of the shaking table proprietary controller are activated causing automatic termination of the test in order to prevent the damage that might otherwise ensue. This result are contrary to conventional wisdom that the larger the damping ratio, the more stable the substructured system.

**Figure 4.15. Reaction force of System 1.**

**Figure 4.16. Shaking table displacement for System 2.**
A further test was conducted using the same test system (System 2) but without any outer loop control. The shaking table response in Figure 4.17 shows that in this case the substructured system is stable, although clearly accuracy is of concern due to the uncompensated for delay of the transfer system. Comparison of Figure 4.16 and Figure 4.17 indicates that, once again contrary to conventional wisdom, instability in this system is induced by the delay compensation algorithm.

![Shaking table displacement for System 2 without controller.](image)

**Figure 4.17.** Shaking table displacement for System 2 without controller.

### 4.2 STABILITY ANALYSIS FOR REAL-TIME SUB-STRUCTURING

The dynamic substructure testing method has been recognized as prone to instability and inaccuracy as a result of the dynamics of the transfer system, the physical substructure and the numerical substructure. To reduce the risk of damage to the physical substructure and the transfer system, it is desirable to obtain a feasibility estimate for a proposed substructure test before the test is attempted. Hence stability takes precedent over accuracy. Additionally, reliable estimates of the stability of a substructured system are required in order to evaluate the relative performance of different (outer loop) control methodologies.

Horiuchi et al. (1999) took the lead in the investigation of the stability of actuator-based substructuring. An energy balance approach was used to calculate the critical time delay of a SDOF system wherein a damper was used as the physical substructure. Results showed that the
time delay of transfer system was equivalent to a negative damping and that the system became unstable when the negative damping exceeded the damping of the physical substructure. Wallace et al. (2005a) and Kyrychko et al. (2006) also obtained the exact expressions of the critical time delay using a delay differential equation. Wu et al. (2006) investigated the stability of a SDOF system with constant delay using an amplification matrix approach. In these works, the dynamics of transfer systems were all approximated as a time delay, which is equivalent to a phase lag proportional to frequency. However as previously seen, the model for the shaking table is a high order system wherein the phase lag is not proportional to frequency. Additionally, a significant magnitude error exists. This section focuses on the development of an analytical method to estimate the critical point of a substructured system based on the shaking table model developed in §4.1.

4.2.1 Root locus method for substructuring tests

Root locus analysis (Richard & Robert (2002)) is a graphical method for examining how the roots of a system change with variation of a certain system parameter, commonly the gain of a feedback system. In addition to designing for the damping ratio and natural frequency of a feedback system, the root locus can be used to determine the stability of the system. In addition to the transfer system dynamics, the physical and numerical parameters of the substructured system (i.e. mass, damping and stiffness) also have an effect on the stability. For a certain substructured system, the sensitivity of the roots of the system to a parameter of the system can be analysed using root locus when the other parameters are fixed. Mercan et al. (2008) and Chi et al. (2011) investigated the effect of time delay on the real-time dynamic testing using root locus technique.

The dynamic performance of a closed-loop control system is described by the closed-loop transfer function:

\[
T(s) = \frac{y(s)}{r(s)} = \frac{p(s)}{q(s)}
\]  

(4.38)
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Where \( r(s) \) and \( y(s) \) are the input and response of the system respectively, \( p(s) \) and \( q(s) \) are polynomials in \( s \). The roots of the characteristic equation \( q(s) \) determine the stability of the system. \( q(s) \) can be rearranged as the multiplying factor in the form:

\[
q(s) = 1 + KG(s) \tag{4.39}
\]

Where \( K \) is a variable parameter, \( G(s) \) is the polynomial in the form of poles and zeros. The characteristic roots of the system must satisfy the following equation:

\[
1 + KG(s) = 0 \tag{4.40}
\]

When one of the roots lies in the right hand side of the S-plane, the system is unstable, and only if one of the roots lies on the imaginary axis (the pure imaginary roots), the system is critically stable. Therefore, the value of \( K \) corresponding to the pure imaginary roots is the critical point of the system. MATLAB’s \textit{rlocus} command supplies a convenient numerical technique to acquire the critical stability point as \( K \) is increased from zero to infinity.

Rewriting Equation 4.32 gives the closed-loop characteristic equation (CLCE) of the substructured system using the shaking table:

\[
1 + G_{st}G_{sy}G_p = 0 \tag{4.41}
\]

Rearranging Equation 4.41 to be the standard form of root locus, the effect of the parameter of interest (mass, damping, or stiffness) on the stability of the substructured system can be investigated.

\[
1 + K \frac{\text{num}(s)}{\text{den}(s)} = 0 \tag{4.42}
\]

### 4.2.2 Comparative study

To investigate the ability of the root locus technique to provide a stability prediction of a substructured system, a comparative study of a two DOF substructured system is presented.
4.2.2.1 **Numerical prediction using the root locus method**

For a convenient expression, the shaking table transfer function developed in §4.1 is rewritten in polynomial form:

\[
G_{ST}(s) = \frac{n_0}{s^4 + n_1s^3 + n_2s^2 + n_is + n_0}
\] (4.43)

where \(n_i\) are the polynomial coefficients of variable \(s\). After substituting Equation 4.43, Equation 4.35 and Equation 4.36, into Equation 4.41, the CLCE of the system can be expressed as:

\[
1 + \frac{n_0m_p(c_p + k_p)s^2}{(s^4 + n_3s^3 + n_2s^2 + n_is + n_0)(m_Ns^2 + c_Ns + k_N)(m_ps^2 + c_ps + k_p)} = 0
\] (4.44)

where \(m_N, c_N, k_N\) are the mass, damping and stiffness of the numerical substructure, and \(m_p, c_p, k_p\) are the mass, damping and stiffness of the physical substructure. Taking the ratio between the mass of physical substructure and the mass of the numerical substructure as the variable parameter of this system and reshaping this equation, the standard form required for a roots locus analysis is produced:

\[
1 + \frac{m_p}{m_N} \frac{n_0\left(2\zeta_p\omega_p^2 + s^2\right)}{s^4 + n_3s^3 + n_2s^2 + n_is + n_0\left(s^2 + 2\zeta_N\omega_N^2 + \omega_N^2\right)(m_ps^2 + 2\zeta_p\omega_p^2 + \omega_p^2)} = 0
\] (4.45)

where \(\omega_N, \xi_N, \omega_p, \xi_p\) are the natural frequency and damping ratio of the numerical and physical substructure respectively. It is clear that the \(m_p/m_N\) parameter varies from zero to infinity, making the range of parametric variation of the system too big to practically describe a curve. Alternatively, another variable is defined:

\[
\sigma = \frac{m_p}{m_p + m_N}
\] (4.46)
where $\sigma (\sigma \in [0,1])$ is the ratio between the physical mass and the summation of the physical and numerical mass. Substituting Equation 4.46 into Equation 4.45, the CLCE with variable parameter $\sigma$ is obtained:

$$1+\sigma\left[\frac{n_0 \left(2\xi_p s + \omega^2_p\right)s^2}{\left(s^2 + n_3 s^3 + n_2 s^2 + n_1 s + n_0\right)\left(s^2 + 2\xi_n s + \omega^2_n\right)\left(m_p s^2 + 2\xi_p s + \omega^2_p\right)} - 1\right] = 0$$

(4.47)

Here mass ratio ($\sigma$) is regarded as the criterion for the assessment of the stability of the substructured system. As mass ratio increases, the roots of CLCE will migrate towards the imaginary axis. The critical mass ratio is defined as the mass ratio where the roots of the CLCE meet the imaginary axis.

### 4.2.2.2 Experimental verification

The experiments in this section focus on the verification of the roots locus method for shaking table substructuring feasibility assessment. The adopted test configuration is as in §4.1.5 wherein both the numerical and the ‘physical’ substructures are modelled numerically; as illustrated in Figure 4.18, only the transfer system exists in the physical world. The numerical response of interface generated from a DSpace real time control board is used as the control signal for shaking table proprietary controller, and the acceleration of shaking table is fed back to the DSpace board to calculate the reaction force.

![Figure 4.18. Experimental set-up.](image)

Two cases are presented to illustrate the validity of roots locus method to forecast the stability boundaries of shaking table substructured systems. The numerical values for the parameters are provided in Table 4.1. In both cases, the physical substructure is taken to be identical to the
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numerical substructure. In Case 1, the frequency is fixed (at 1.94Hz) and the damping variable (0.5-50%). In Case 2, the damping ratio is fixed (at 2%), and the frequency variable (0-10Hz).

Table 4.1. Parameters of 2DOFs system

<table>
<thead>
<tr>
<th>Case ref.</th>
<th>Physical substructure</th>
<th>Numerical substructure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency $\omega_P$ (Hz)</td>
<td>Damping ratio $\xi_P$ (%)</td>
</tr>
<tr>
<td>1</td>
<td>1.94</td>
<td>0.5-50</td>
</tr>
<tr>
<td>2</td>
<td>0-10</td>
<td>2</td>
</tr>
</tbody>
</table>

To experimentally obtain the stability boundary, a series of tests with different mass ratio were conducted. Initially, a mass ratio below the predicted critical stability point was adopted. If the test was successful (i.e. stable), the mass ratio was incrementally increased and the test repeated. This continued until instability occurred. Instability was characterised by the command signal increasing without bound thereby activating the safety mechanisms of the shaking table proprietary controller in order to prevent the damage due to the transfer system that would otherwise ensue. The mass ratio during the last successful test is referred to as the stable point; the mass ratio during the first unsuccessful test is referred to as the unstable point. Figure 4.19 (variable damping) and Figure 4.20 (variable frequency) compares the experimentally measured stable and unstable points (represented by the squares and circles respectively) with the analytical curves resulting from the roots locus method.
Figure 4.19. Comparison of analytical and experimental stable boundaries of Case 1.

Figure 4.20. Comparison of analytical and experimental stable boundaries of Case 2.

Experimentally derived stability points agree closely with the trend predicted by the roots locus analysis. Thus, when the dynamics of the transfer system are properly taken into account, the roots locus technique can be used to assess the stability of substructured system. In the following Chapters the roots locus technique will be used to evaluate the stability boundaries of numerous different controllers developed for substructuring control.
4.2.3 Stability analysis for shaking table substructuring

Roots locus analysis can be used to evaluate the effect on stability of varying the parameters of the physical substructure and the numerical substructure together and independently. Three cases are considered with adopted parameters as indicated in Table 4.2. In Case 3 the parameters of the physical substructure are held constant while the parameters of the numerical substructure vary; in Case 4, the numerical substructure is fixed while the physical substructure varies; in Case 5, both physical and numerical substructures vary together.

The analytical stability boundaries associated with Case 3 (fixed-physical) are presented in Figure 4.21. In general, the stability increases with both $\omega_p$ and $\xi_p$. The higher is the damping, the larger the increase and the rate of increase of stability. However, for lightly damped, low frequency numerical substructures like those of interest in most Civil Engineering applications, the effect $\xi_N$ is comparatively small and the effect of $\omega_N$ is reversed.

Table 4.2. Parameters of 2DOFs system

<table>
<thead>
<tr>
<th>Case ref.</th>
<th>Physical substructure</th>
<th>Numerical substructure</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Frequency $\omega_p$ (Hz)</td>
<td>Damping ratio $\xi_p$ (%)</td>
</tr>
<tr>
<td>3</td>
<td>1.94</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>0.1-10</td>
<td>0.5/2/5/10/20/30</td>
</tr>
<tr>
<td>5</td>
<td>0.1-10</td>
<td>0.5/2/5/10/20/30</td>
</tr>
</tbody>
</table>

Figure 4.22 presents the analytical results associated with Case 4 (fixed-numerical). Here, stability decreases as $\omega_p$ increases and at rates which increase as $\xi_p$ lessens. The conventional wisdom that the stability increases with damping is valid if $\xi_p < 10\%$. Thereafter, for low magnitudes of $\omega_p$, decreases in stability are measured with increasing $\xi_p$ (a finding also born testament to both analytically and experimentally in Figure 4.19).

Figure 4.23 displays the analytical stability boundaries of Case 5. The trends evident in the figure bear a close resemblance to those of Case 4 (fixed-physical) suggesting that the properties of the
physical substructure have a larger determining influence on the stability of this shaking table substructuring configuration.

Figure 4.21. Stable boundaries of 2DOFs system for condition 3.

Figure 4.22. Stable boundaries of 2DOFs system for condition 4.
A commonly used scheme that is intended to enhance both the stability and the accuracy of substructuring tests is delay compensation. The assumption here is that the transfer system dynamics can be approximated merely by a time delay and compensated for by forward polynomial extrapolation of the numerical substructure displacement by a magnitude equivalent to that delay. For actuator-based substructuring systems (i.e. not using a shaking table) numerous different implementations of delay compensation have been proposed which vary from computationally undemanding single step prediction techniques to the more complex adaptive schemes intended to deal with time varying delays (Darby et al (2002), Wallace et al (2005b)).

Horiuchi et al. (2001) demonstrated that the stability of delay compensation depends on the time/phase lag of transfer system and the frequency of substructured system. Successful application of delay compensation can result only if the transfer system has a small delay and the frequency bandwidth of the substructuring system is low. The large mass of shaking tables ensures that their response is slower than that of standalone actuators. Until now, the applicability of delay compensation techniques to shaking table substructuring has not been reported.
4.3.1 Frequency domain analysis of delay compensation

In delay compensation the dynamic characteristics of transfer system are assumed to be a pure delay. The displacement achieved by the transfer system ($y_{IP}$) is equivalent to the desired displacement output by the numerical substructure ($y_{IN}$) delayed by $\tau$ seconds:

$$y_{IP} = y_{IN}(t - \tau) \quad (4.48)$$

To synchronise the desired displacement with the achieved displacement, the desired displacement at $\tau$ seconds in the future is predicted and it is the predicted value that is used as the input command signal to the proprietary controller of transfer system. The command signal ($u$) from delay compensation is then:

$$u = \sum_{i=0}^{m} a_i y_{IN}(t - i\tau) \quad (4.49)$$

where $a_i$ are the polynomial coefficient of $y_{IN}$ for the forward prediction, $y_{IN0}$ is the present calculated displacement of the interface, $y_{INi}$ are calculated displacements of the interface at previous time increments $i\tau$, and $\tau$ is the time delay of the transfer system.

Through the Laplace transform, the transfer function of Equation 4.49 can be given:

$$\begin{align*}
    u(s) &= H_c(s)y_{IN}(s) \\
    H_c(s) &= \sum_{i=0}^{m} a_i e^{-is}\tau
\end{align*} \quad (4.50)$$

To analyse the performance of substructured system in frequency domain it is necessary to rationalise this expression using an approximation. The Padé approximation (Golub & Loan (1996)) can be used:

$$H(s) \approx \frac{\sum_{j=0}^{n} (-1)^j k_j s^j}{\sum_{j=0}^{n} k_j s^j} \quad (4.51)$$
where \( n \) is the order of the approximation. The coefficients \( k_i \) are functions of \( n \). Substituting Equation 4.51 into Equation 4.50 gives the rational transfer function of the delay compensation controller:

\[
H_{DC}(s) = \sum_{i=0}^{n} a_i \frac{(-1)^i k_i s^i}{\sum_{j=0}^{n} k_j s^j}
\]

(4.52)

### 4.3.2 Stability analysis

To investigate the improvement that the delay compensation method can bring to shaking table substructuring systems several case studies of a two DOF substructured system are presented. The commonly used third-order polynomial extrapolation scheme is chosen for analysis, coefficients of which are \( a_0 = 4, a_1 = -6, a_2 = 4, a_3 = -1 \).

#### 4.3.2.1 Comparative study

Substituting Equation 4.52 into Equation 4.47, the CLCE for the substructured system based on delay compensation control can be obtained:

\[
1 + \sigma \left[ H_{DC}(s) \frac{n_0 \left( 2 \xi_p s + \omega_p^2 \right) s^2}{\left( s^4 + n_3 s^3 + n_2 s^2 + n_1 s + n_0 \right) \left( s^2 + 2 \xi_N s + \omega_N^2 \right) \left( m_p s^2 + 2 \xi_p s + \omega_p^2 \right) - 1} \right] = 0
\]

(4.53)

The roots locus analysis developed in §4.2.1 can be used to obtain the stability boundaries for the delay-compensated shaking-table-substructuring system described by Equation 4.53. MATLAB’s `pade` command is used to estimate the coefficients of Padé approximation for the time-delay; an 8th order approximation is used. The (constant) time delay (\( \tau \)) of the shaking table is estimated to be 28.5 milliseconds.

The two cases described in §4.2.2 are reanalyzed numerically and experimentally. To recap, both the numerical and the ‘physical’ substructures were modelled numerically and only the transfer system existed in the physical world. The experimental set-up is that shown in Figure 4.18.
Figure 4.24 and Figure 4.25 show the experimentally measured stable and unstable points (represented by the squares and circles respectively) alongside the analytical stability boundaries of the substructured system both when delay compensation (DC) is used and when no additional outer-loop controller is used (NC).

Figure 4.24. Comparison of stability boundaries of Case1.

Figure 4.25. Comparison of stability boundaries of Case2.
The close correlation between the experimental stability points and the analytical curves illustrates that the analytical method can correctly assess the stability of delay compensation. The comparison of the two analytical curves in Figure 4.24 shows that, for the variable damping system of Case 1, delay compensation enhances the stability (compared to the no additional controller case) only when the system is lightly damped. For the variable frequency system of Case 2 (Figure 4.25) the delay-compensation offers significant stability enhancement only at low frequencies.

Furthermore, the test data previously presented in §4.1.5 are evaluated in the light of the presented stability analysis and re-plotted using asterisks in Figure 4.24. The mass of the physical and numerical substructures were 56.38kg and 84.57kg respectively, giving a mass ratio \((\sigma)\) (for this data set) equal to 0.6. When the damping ratio is 5% the test is below the DC stability boundary but above the NC stability boundary – System 1 is stable only when using DC. However, when the damping ratio is 25%, the situation is reversed and the test falls below the NC stability boundary, but above the DC stability boundary. System 2 is unstable using DC, but stable when no additional controller is used.

### 4.3.2.2 The influence of substructure properties on delay compensation stability

In §4.3.2.1 the effects on stability of varying the physical and numerical substructures were analysed when no outer-loop controller was used. Below, identical test configurations are analysed but when delay compensation is used. The no-controller configuration is signified by the abbreviation NC, delay compensation by DC.

The analytical stability boundaries associated with Case 3 (physical substructure constant with \(\omega_p = 1.94\text{Hz}\) and \(\xi_p = 2\%\), numerical model variable) are presented in Figure 4.26. Unlike for the NC case, increasing \(\omega_N\) has a detrimental effect on the stability of the DC system. The frequency at which the NC and DC curves intersect is defined as the cross-over frequency. The lower is the damping, the bigger is the cross-over frequency. Delay compensation enhances the stability if \(\omega_N\) is less than cross-over frequency. Otherwise, delay compensation diminishes stability.
Figure 4.26. Stability boundaries of the Case 3 delay-compensated shaking-table substructuring system.

Figure 4.27 displays the boundaries of Case 4 (numerical substructure constant with $\omega_N = 1.94$Hz and $\xi_N = 2\%$, physical model variable). DC offers a stability improvement over NC only when both the frequency and the damping are low. Furthermore, although the NC system responds according to conventional wisdom and exhibits increased stability with increased damping, the low-frequency stability of the DC system deteriorates as the damping increases.

Similar observations can be made for Case 5 (physical substructure and numerical substructure variable with $\omega_P = \omega_N$ and $\xi_P = \xi_N$) using Figure 4.28. Like the NC system, the properties of the physical substructure are seen have a dominant influence on the stability of the DC system. Stability enhancements brought about by DC are apparent only at the low frequency/damping end.
Figure 4.27. Stability boundaries of the Case 4 delay-compensated shaking-table substructuring system.

Figure 4.28. Stability boundaries of the Case 5 delay-compensated shaking-table substructuring system.
4.3.3 Transfer system accuracy

The ultimate aim of any substructuring test is to reproduce the response of a system as if it were being tested not as substructured parts but conventionally, in its entirety. To achieve this goal, stability is necessary but not sufficient; attention must also be given to accuracy. Here, the accuracy of shaking table delay compensation is assessed in terms of the similarity between the intended and the measured motions undergone by the transfer system.

A transfer function between the achieved displacement $y_{IP}$ and the desired displacement $y_{IN}$ can be obtained by combining the transfer functions of the shaking table and the delay compensation:

$$G_{DC} = \frac{y_{IP}(s)}{y_{IN}(s)} = H_{DC}(s)G_{ST}(s)$$  \hspace{1cm} (4.54)

The corresponding expression for the shaking table system with no outer loop controller is provided by Equation 4.24.

A 0.1-10Hz sine sweep excitation signal is utilised to evaluate the accuracy of shaking table substructuring using delay compensation. The following test parameters are adopted: $m_N = m_P = 56.38\text{kg}$, $\omega_N = \omega_P = 1.936\text{Hz}$, and $\xi_N = \xi_P = 0.05$. Analytical and experimental results are presented in the frequency domain in Figure 4.29(a) and Figure 4.29(b) respectively. The presented transfer functions are measured between the desired displacement output from the numerical substructure and the achieved displacement undergone by the transfer system. Accuracy demands a transfer function magnitude of unity and phase of zero. For contrast, Figure 4.29 also presents the shaking table transfer function (measured between the displacement command signal and displacement response).

Both the analytical and experimental results show that delay compensation can rectify the shaking table phase lag (apparent from 0Hz and otherwise ruinous for accuracy) only when the excitation frequency is low (i.e. less than 4Hz in this case). As the excitation frequency increases above this value, the desired response is amplified and phase leads (between 4Hz and 6Hz) then lags (above 6Hz) occur. The poor performance of delay compensation at higher frequencies is illustrated in the time domain in Figure 4.30.
Figure 4.29. Comparison of shaking table performance in frequency domain.

Figure 4.30. Comparison of delay compensated shaking table performance in time domain.
The performance of delay compensation is insufficient to employ the parameter set defined to be of interest by the SERIES project (see §1.1) in particular when the main period of the fixed base structure is 0.2 seconds. It is thereby deemed necessary to develop a new controller that is functional at higher frequencies.

4.4 FULL STATE CONTROL VIA SIMULATION

In previous sections it has been shown that the stability and the accuracy of a substructuring test are determined by the dynamic characteristics of the substructures – physical and numerical – and by those of the transfer system – the shaking table and its controller. Additionally, the performance of the transfer system has been demonstrated to result from the physical, mechanical, hydraulic and dynamic characteristics of its sub-components. When a transfer system takes the form of a servo-hydraulic actuator, its dynamics can be reduced to a pure time delay and nulled by delay compensation. However, the frequency-dependent magnitude and phase errors that exist for more complex transfer systems such as shaking tables are beyond the scope of delay compensation.

Much attention has recently been focused on developing mathematical models to represent the dynamics of servo-hydraulic systems (Blondet & Esparza (1988), Conte & Trombetti (2000), Williams et al. (2001), Zhao et al (2003)). If such models are inverted, they can conceivably be used to cancel the magnitude and phase errors of a transfer system (Carrion et al (2007), Carrion et al (2009)). In this section, this conception is used to develop anew controller for substructuring. Based on the Inverse Dynamics Compensation via Simulation, and using full-states feedback control, the new substructuring controller is called Full State Compensation via Simulation (FSCS). Below, the FSCS controller is introduced and its performance is verified experimentally.

4.4.1 Background

In this section, some background into the development of FSCS is presented. An overview of inverse dynamics control is followed by an introduction to the concepts of Inverse Dynamics Compensation via Simulation and full states feedback.
4.4.1.1 Inverse dynamics control

Figure 4.31 shows the block diagram of an open-loop Inverse Dynamics Control (IDC) system. In the figure, the plant (i.e., an actuator or shaking table) is represented by $G_{\text{Plant}}(s)$, the transfer function model of the plant is written as $G_{\text{Model}}(s)$, and the IDC is denoted by $G_{C}(s)$. The reference, plant-output, and inverse-dynamics control signals are noted by $r$, $y$ and $u_c$, respectively.

![Diagram of IDC system]

**Figure 4.31. Open-loop inverse-dynamics control.**

IDC is based on a linear inverse model of the underlying plant dynamics, which are parameterised via system identification. A good level of parametric certainty is essential.

The IDC control law is written as:

$$G_{C}(s) = G_{\text{Model}}^{-1}(s)$$  \hspace{1cm} (4.55)

When the parameters are known exactly, the plant output $y$ is given by:

$$y(s) = G_{\text{Plant}}(s)G_{C}(s) r(s) = r(s)$$  \hspace{1cm} (4.56)

Hence, feed forward IDC nulls the dynamics of the plant, resulting in a compensated system with a magnitude of one and a phase of zero. Given:

$$G_{\text{Model}}(s) \equiv G_{\text{Plant}}(s)$$  \hspace{1cm} (4.57)

then:
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\[ G_{Plant}(s)G_c(s) = 1 \quad (4.58) \]

As seen in Section 4.1.2, the shaking table transfer system has several poles and no zeros. The general shaking table model written in polynomial form is:

\[ G_{ST}(s) = \frac{B}{s^n + \sum_{i=1}^{n} b_{n-i}s^{n-i}} \quad (4.59) \]

where \( B \) is the gain, and \( b_i \) are the polynomial coefficients of variable \( s \), which can be rewritten as:

\[ G_{ST}(s) = \frac{B}{\prod_{i=1}^{n} (s - \lambda_i)} \quad (4.60) \]

where \( \lambda_i \) are the poles of the system, and \( n \) is the order of the system. The inverse model is thus:

\[ G^{-1}_{ST}(s) = \frac{\prod_{i=1}^{n} (s - \lambda_i)}{B} \quad (4.61) \]

Equation 4.61 is an improper transfer function with purely derivative action. Implementation of IDC in this form would promote instability and inaccuracy by amplifying high-frequency signal components that may be associated with either electrical noise or unmodelled/uncertain system dynamics. To avoid this, closed-loop compensation strategies with unit gain low-pass filters have been implemented to ensure stability and robustness (Carrion (2007), Carrion (2009)). Finally, it should be reiterated that a linear inverse model has to be used in this controller; nonlinearity cannot be accommodated.

The performance of IDC has been demonstrated in various works (eg. Nakanishiet al(2007) and Zhou et al.(2006)).
4.4.1.2 Inverse dynamics compensation via simulation

Inverse Dynamics Compensation via Simulation (IDCS) uses a parallel, real-time simulation loop of the plant to generate a noise-free, inverse dynamics control signal, $u_c$. Tagawa & Fukui (1994) introduced the concept of IDCS, demonstrated its applicability for non-linear systems, and successfully implemented IDCS within the control of both a servohydraulic actuator with a geometric nonlinearity (Tagawa et al (2010), Tu et al (2010)) and a full-scale substructuring shaking table test (Ji et al (2009)).

Figure 4.32 illustrates the scheme of open-loop IDCS wherein the numerical model of the plant and a controller are denoted by $G_{model}$ and $G_{CS}$ respectively. One key feature of open-loop IDCS is that the control gain of $G_{CS}$ can be set to a very high value in the simulation environment without the danger of noise amplification since the plant-output signal originates from $G_{model}$ and not the plant itself.

![Figure 4.32. Block diagram of a real-time IDCS loop.](image)

Figure 4.33 illustrates the scheme of closed-loop IDCS wherein the plant is also included in the control loop.

![Figure 4.33. Block diagram of a real-time, closed-loop IDCS technique.](image)
If $G_{Model}$ is a close approximation of $G_{Plant}$, $u_s$ (generated from $G_{CS}$) can, in principle, control the plant perfectly. Inevitably, however, parametric variations and uncertainties have an impact on performance. In practice, stability and robustness are provided by implementing a closed loop feedback controller as seen in Figure 4.33.

### 4.4.1.3 Full state feedback control

Full State Feedback (FSF) control cannot be introduced without a consideration of the state space equations of a linear time invariant (LTI) system:

\[
\begin{align*}
\dot{x} &= Ax + Bu \\
y &= Cx
\end{align*}
\]  

(4.62)

Here: $\mathbf{x}$ is the state vector, $\dot{\mathbf{x}}$ is the state derivative vector, $\mathbf{u}$ is the control signal, $\mathbf{y}$ is output signal, $\mathbf{A}$ is the state transition matrix, $\mathbf{B}$ is the input transition vector, $\mathbf{C}$ the state observer vector. FSF control can then be depicted as in Figure 4.34.

![Figure 4.34. Block diagram of full state feedback control](image)

The control action is achieved by introducing feedback matrices, $\mathbf{K}$, to produce the control input $u(t)$:

\[
u(t) = -\mathbf{K}\mathbf{x}(t) + r(t)
\]

(4.63)

The closed-loop system poles can hence be positioned in pre-determined locations by selecting appropriate feedback matrices (Richard & Robert (2002)).
FSF control requires knowledge of the state variables of the plant. In practice, however, only the displacement and acceleration of the plant are usually measured. While it may be possible to derive the plant velocity from existing measurements (Stoten (2001)), the state variables that are required to control higher order systems (i.e. in excess of three) are unavailable.

### 4.4.2 Design of full state control via simulation for substructuring

Full State Control via Simulation (FSCS) is a conjunction of IDCS and FSF (IDCS-FSF) that has been designed to capitalise on the advantages of its forbearers while overcoming their inherent deficiencies. This section describes the development of FSCS from IDCS and FSF and indicates the different configurations of that FSCS can adopt when applied to shaking table substructuring.

#### 4.4.2.1 ICDS-FSF design

Figure 4.35 illustrates the detail of a FSF controlled shaking table. The transfer function of shaking table based on FSF (STFSF) is given by:

\[
G_{STFSF} = \frac{y_s}{r} = \frac{k_f B}{s^n + \sum_{i=1}^{\infty} (b_{n-i} + Bk_{n-i})s^{-i}}
\]  

(4.64)

![Diagram of IDCS-FSF system](image)
The frequencies \( \omega_i \) corresponding to the poles \( \lambda_i \) of the shaking table model limit its operational bandwidth. The frequencies associated with the shaking table used in this work are \( \omega_1 = 9.62\text{Hz}, \omega_2 = \omega_3 = 10.01\text{Hz}, \omega_4 = 48.57\text{Hz} \).

Using FSF control, the shaking table can be reconfigured as an \( n \)th-order system comprised of \( n \) identical first-order systems with poles located at a pre-defined optimum frequency (Figure 4.36). Defining \( \omega_{\text{opt}} \) as the frequency to which the poles of the shaking table are forced by FSF, the transfer function of the optimised shaking table is:

\[
G_{\text{opt}} = \frac{\omega_{\text{opt}}^n}{(s + \omega_{\text{opt}})^n}
\]

(4.65)

Using binomial expansion theorem this expression can be rewritten:

\[
G_{\text{opt}} = \frac{\omega_{\text{opt}}^n}{s^n + \sum_{i=1}^{n} \frac{n!}{i!(n-i)!} \omega_{\text{opt}}^i s^{n-i}}
\]

(4.66)

Setting the polynomial coefficients of Equation 4.64 equal to Equation 4.66 gives the following expressions which can be used to obtain the \( n \) coefficients:

\[
\begin{align*}
(b_{n-i} + Bk_{n-i}) &= \frac{n!}{n!(n-i)!} \omega_{\text{opt}}^i, (i = 1, \ldots, n) \\
Bk_f &= \omega_{\text{opt}}^n
\end{align*}
\]

(4.67)

![Figure 4.36. Frequency characteristics of shaking table.](image)
4.4.2.2 Open-loop FSCS design

From Figure 4.35, the IDCS-FSF transfer function between the excitation \( r \) and the noise-immune control signal \( u \) is seen to be equivalent to that for IDC(§4.4.1):

\[
G_{IDCS-FSF} = \frac{u}{r} = G_{ST}^{-1}G_{opt} \tag{4.68}
\]

As seen in Figure 4.37, taking the signal \( u \) generated from IDCS-FSF as the control signal of shaking table gives an open-loop form of FSCS (oFSCS) with transfer function:

\[
G_{OFSCS} = G_{Table}G_{IDCS-FSS} \tag{4.69}
\]

The performance of oFSCS is determined by the closeness of fit between the transfer function of the shaking table, \( G_{Table} \), and that of the numerical model of the shaking table used inside \( G_{IDCS-FSF} \) and \( G_{ST} \). As such, system identification errors, parametric variations, noise and uncertainty all have an impact on the performance of oFSCS. To minimise these impacts, a feedback control loop is required.

\[ r(t) \quad \text{IDCS based on FSF} \quad G_{IDCS-FSF} \quad u_s \quad \text{Shaking table} \quad G_{Table} \quad y \]

Figure 4.37. Open-loop FSCS.

4.4.2.3 Closed-loop FSCS design

When a fixed-gain, proportional feedback controller \( k_g \) is adopted as in Figure 4.38, the closed-loop FSCS (cFSCS) transfer function is seen to be:

\[
G_{CFSCS} = \frac{G_{Table}(G_{IDCS-FSS} + k_g)}{1 + k_g G_{Table}} \tag{4.70}
\]
Here, $G_{\text{Table}}$ represents the transfer function of the real shaking table, not the model. Combining cFSCS control with the shaking table substructuring system produces the novel control strategy for substructuring illustrated in Figure 4.39.

**Figure 4.38.** Closed loop FSCS.

**Figure 4.39.** Full state control via simulation for substructuring.

### 4.4.3 Practical application of FSCS

Two models of the shaking table have previously been described (§4.1.3). The 3rd and 4th order model of shaking table used for FSCS design are shown in Equation 4.24 and Equation 4.25 respectively. The operational bandwidth of the third-order model of shaking table has been established to be 7Hz; the fourth-order model, 20Hz. It is these bandwidths that set that of the FSCS controller. Hence, different variants of FSCS control (3rd order, 4th order) can be used in either open- or close-loop. Here set noFSCS and ncFSCS represent the $n$-order open- and close-
loop FSCS controllers, where \( n \) indicates the order of the shaking table model used in the FSCF design, and the ‘o’ and ‘c’ appendage indicate open- and close-loop respectively.

In theory, the higher the magnitude of \( \omega_{opt} \) the better the accuracy of FSCS. However, magnitude limitations on \( \omega_{opt} \) are imposed by the characteristics of the shaking-table models outside of their operational bandwidth in conjunction with the high frequency response of the physical components of the substructuring system and by the level of electrical noise in the feedback signals (\( f_{reaction} \) and \( y_{IP} \)). For the tests considered herein, appropriate magnitudes for \( \omega_{opt} \) were determined experimentally for the 3\(^{rd}\) and 4\(^{th}\) order models to be 150Hz and 200Hz respectively. The third and forth order inverse shaking table transfer functions with these magnitudes of \( \omega_{opt} \) are presented in Figure 4.40. While the 4\(^{th}\) order model offers a wider operational bandwidth, it is clear from this Figure 4.that it is also associated with additional gain outside of its operational bandwidth.

When the substructure and excitation frequencies are low, it is sufficient to use the 3\(^{rd}\) order FSCS. If the system is associated with high frequencies, better accuracy may be achieved with the 4\(^{th}\) order FSCS. However, since both shaking table models are associated with amplification at higher frequencies which, if high frequency components exist in the feedback systems, may work to push the system towards inaccuracy or instability.

![Figure 4.40. Inverse models of the shaking table from (a) 0-10Hz and (b) 10-100Hz.](image-url)
4.4.4 Stability analysis

To investigate the improvement that FSCS can bring to shaking table substructuring systems several now-familiar case studies of a two DOF substructured system are presented below.

4.4.4.1 Comparative study

Substituting Equation 4.70 into Equation 4.47, the CLCE for the substructured system based on FSCS can be obtained:

\[
1 + \sigma \left[ G_{SFSCS} \frac{(2\xi_Ps + \omega_N^2)s^2}{(s^2 + 2\xi_Ns + \omega_N^2)(m_ps^2 + 2\xi_Ps + \omega_P^2)} - 1 \right] = 0 \tag{4.71}
\]

Roots locus analysis can be used to obtain the stability boundaries for the FSCS-controlled shaking-table-substructuring system described by Equation 4.71.

The two cases described in Table 4.1 of §4.2.2 are reanalyzed numerically and experimentally. To recap, both the numerical and the ‘physical’ substructures were modelled numerically and only the transfer system existed in the physical world. The experimental set-up is that shown in Figure 4.41. For Case 1, the error feedback gain \( k_e \) was set to 0.5 and 4th of FSCS was adopted. For Case 2, the error feedback gain \( k_e \) was again set to 0.5 but 3rd of FSCS was adopted.

---

![Figure 4.41. Experimental set-up.](image-url)
Figure 4.42 and Figure 4.43 show the experimentally measured stable and unstable points (represented by the squares and circles respectively) alongside the analytical stability boundaries of the substructured system when FSCS, delay compensation (DC), and no-additional outer-loop controller (NC) are used.

In Figure 4.43, the close agreement of the experimental stability points and the analytical curve illustrates that the analytical method can correctly assess the stability of FSCS. The analytical curve in Figure 4.42 does not match with the experimental stability points quite so well, probably due to uncertainties associated with error feedback. However, the analytical curve still provides a reasonable stability prediction. The comparison of the analytical curves in Figure 4.42 and Figure 4.43 shows that FSCS gives a good enhancement to the stability, outperforming both NC and DC.

![Comparison of stable boundaries of Case 1.](image)
Figure 4.4.3. Comparison of stable boundaries of Case 2.

4.4.4.2 The influence of substructure properties on FSCS stability

Previously, the effects on stability of varying the physical and numerical substructures were analysed for both NC and the DC. Below, the same test configurations (detailed within Table 4.2 of §4.2.3) are analysed but when 4thoFSCS is used.

The analytical stability boundaries associated with Case 3 (physical substructure constant with $\omega_p = 1.94\text{Hz}$ and $\xi_p = 2\%$, numerical substructure variable) are presented in Figure 4.44. It shows that all the six critical stability curves of 4thoFSCS are near to one, which illustrates that the properties of numerical substructures do not affect the stability of the FSFC-controlled shaking-table substructuring system.

Figure 4.45 displays the boundaries of Case 4 (numerical substructure constant with $\omega_N = 1.94\text{Hz}$ and $\xi_N = 2\%$, physical model variable). 4thoFSCS works to stabilise all of the systems and offers dramatic improvements over both NC and DC. An exception occurs at low magnitudes of $\xi_p$ where DC offers a stability advantage over 4thoFSCS. The reason for this is the small time/phase lag that is apparent in the 4thoFSCS system due to the magnitude limitations imposed upon $\omega_{opt}$ (§4.4.3). Here, with $\omega_{opt} = 200\text{Hz}$, a time lag of around 3ms exists. While such a small delay is negligible for typical civil engineering substructuring systems, it becomes influential for lightly
damped systems. Delay compensation, on the other hand, is capable of compensating for the phase lag completely in the low frequency range.

Figure 4.44. Stability boundaries of the Case 3 FSFC-controlled shaking-table substructuring system.

Figure 4.45. Stability boundaries of the Case 4FSFC-controlled shaking-table substructuring system.
Equivalent conclusions can be drawn from Figure 4.46 for Case 5. The properties of the physical substructure are once again seen to have a larger determining influence on the stability of shaking table substructuring systems.

### 4.4.5 FSCS implementation in an authentic substructuring system

To assess the performance of the new shaking table substructuring controller, an authentic substructuring system with a physical substructure existing the physical world is fabricated and tested. The physical substructure consists of the single DOF oscillator shown diagrammatically in Figure 4.47. It comprises of a bearing carriage that is guided horizontally by a pair of steel rails. The rails terminate in end-blocks secured to a rigid support frame. The bearing-carriage/rail/end-block assembly are precision manufactured components. The self weight of the bearing carriage is 4.43kg which provides a minimum oscillator mass. The oscillator mass can be increased by securing dead weight to the bearing carriage. Fastened between carriage and end block, and enveloping each free length of rail, is a compression spring (four springs total). The springs are configured so that their first and last four coils are closed. This feature allows the spring ends to be screwed into aluminium nuts. To secure the spring in position, the aluminium
nut at one end is secured to the carriage, and that at the opposite end is secured to the end block. The connection between the nuts and the carriage/end blocks is via cap head screws. The oscillator stiffness is thus an independent variable and can be altered by installing springs with different stiffness characteristics.

The oscillator is supported by a rigid support frame constructed from welded aluminium square box section. In the long direction, X-bracing is employed to enhance stiffness. In the cross direction, H frames allow the oscillator height to be an independent variable which can be set at 85.8mm, 219mm, 339.8mm, 473mm or 593.8mm. 1mm thick aluminium shear panels are used to enhance the shear stiffness of the H frames and four rail-insertion holes through each shear panel are required for assembly. The frame is secured down via four aluminium angle footings welded to the floor beams. The physical substructure is designed to offer independent control over the stiffness, mass and lever arm of the oscillator.

Figure 4.47. The physical substructure.
The interaction force imposed by the physical substructure on the numerical substructure is measured using a six degree of freedom force platform (AMTI Inc. OR6-7-2000) that is securely fastened to the shaking table, underneath, and the physical model, above. It is the physical characteristics of this device which determined the limiting footprint and mass of the physical substructure. It was deemed that a maximum of 50kg could be added to the deadload of the oscillator without imposing excessive risk to the force platform giving a maximum oscillator mass of around 55kg. (Note that oscillator mass was intentionally small in relation to the 6 tonne mass of the shaking table platform in order to prevent table-specimen interaction that would add to the challenge of shaking-table RTDS implementation.) The physical substructure is shown attached to the force platform on the shaking table in Figure 4.48. Figure 4.49 shows schematically the implementation of a two DOF substructured system using the physical substructure working in conjunction with the FSCS controller.
4.4.5.1 Demonstration tests

To illustrate the capabilities of FSCS for shaking table substructuring a test is conducted that would not be possible using delay compensation. The adopted system parameters are $m_p = 56.38\text{Kg}, m_N = 141\text{Kg}, \omega_N = 5\text{Hz}, \omega_P = 4.88\text{Hz}, \zeta_N = 0.1,$ and $\zeta_P = 0.025.$ The location of the test in relation to the stability boundaries for the various controllers is presented in Figure 4.50. The displacement record of the El Centro earthquake (NS component of the 1940 El Centro, Station: 117 El Centro Array #9, Component 270°) was chosen to be the reference exciting signal. The 3rd open-loop FSCS was used in this case, the expected frequency of which is 150Hz.

**Figure 4.50. Demonstration test stability**
Figure 4.51 displays the desired and achieved displacement of shaking table and the command signals generated from FSCS controller. It is seen that FSCS controller improves the performance of shaking table and reproduces the interface displacement successfully. The response of the substructuring test (in terms of physical substructure displacement and reaction force) is compared to the response of a two DOF numerical simulation in Figure 4.52. As can be seen, close agreement of both displacement and reaction force is achieved between test and simulation. FSCS substructure testing adequately replicates the response of the complete system.

![Figure 4.51. Shaking table displacement during FSCS demonstration substructuring test.](image)
Figure 4.52. Substructured and simulated response of the physical substructure.

4.4.5.2 Stability

§4.4.4 considered the stability of shaking table substructuring with FSCS when both the numerical and the ‘physical’ substructures are modelled numerically and a close match was observed between experimental and analytical data. Here, the stability of FSCS for the authentic shaking table substructuring system is considered. The parameters used are: \( \xi_N = 0.02, \xi_p = 0.025 \), \( \omega_h = 4.88 \text{Hz} \), \( \omega_N \) taken as an experimental variable with magnitude incremented at 2Hz intervals between 1Hz and 9Hz. The experimental set-up is same as Figure 4.49. 3rd of FSCS is used. The experimentally measured stable and unstable points (represented by the squares and circles respectively) alongside the analytical stability boundaries of the systems are shown in Figure 4.53.
The experimental observed stability points fall well short of the analytical curve. The reason for this shortfall was anticipated in §4.4.3 and can be ascertained by a consideration of the shaking table command signal at the onset of instability (Figure 4.54). The instability is seen to occur at 50Hz, the frequency of transmission of the alternating current electricity supply in the UK which accounts for the main source of electrical noise. Thus, it can be inferred that instability is induced by the noise characteristics of the feedback signals. Other data not presented here suggests that instability might also be associated with the 150Hz resonant frequency of the force plate used to measure the reaction force underneath the physical substructure, or the 70Hz out-of-plane resonant frequency of the springs of the physical substructure.

Figure 4.53. Comparison of stable boundaries.

Figure 4.54. The onset of instability in FSCS substructuring.
4.4.5.3 Transfer system accuracy

The accuracy of FSCS is verified experimentally in both frequency and time domain in this section. To remain within the performance capacities of the table, a new parameter set is employed. The parameters of the tested system are: $\xi_N = 0.25, \xi_p = 0.025$, $\omega_n = 7 \text{Hz}, \omega_r = 4.88 \text{Hz}$, $m_N = 56.38 \text{kg}, m_r = 282 \text{kg}$. FSCS is used and a sine sweep with the frequency range of 0.1 to 10Hz is selected as the excitation. The transfer function of shaking table without additional controller measured from the achieved displacement and command signal is presented as the solid line in Figure 4.55, and the corresponding transfer function of shaking table with FSCS measured from the achieved displacement and desired displacement is presented as dash line.

The experimental results show that FSCS can adequately compensate for both the phase lags and magnitude errors. Additionally, FSCS performs well when the reference excitation attains high frequencies, a fact demonstrated in the time domain in Figure 4.56. A comparison between this figure and the equivalent figure for delay compensation (Figure 4.30) indicates the performance enhancement FSCS is capable of providing.

![Figure 4.55. Comparison of shaking table performance in frequency domain.](image-url)
It is fitting for the substructuring system described in §0 to lend itself to the application of shaking table substructuring to soil structure interaction (SSI) studies where the numerical substructure takes the form of the soil model. This section describes a benchmark test that is then used to validate substructuring test results. A linear lumped-mass soil model is adopted for validation purposes. Following this, a nonlinear macroelement implemented and the effects of nonlinearity on the performance and stability of the shaking table substructuring system are assessed.

4.5.1 The benchmark test

A benchmark shaking table test will be used as a source of experimental data relating to an SSI system in order to validate the substructuring test methodology. The benchmark test was
Shaking table test techniques and fault rupture box testing for SSI

carried out as part of the New Methods of Mitigating the Seismic Risk of Existing Foundations (NEMISREF) project funded by the 5th framework of the European Commission.

As introduced in §2.1, the term ‘shear stack’ refers to the flexible-walled hollow box designed and built to enable the geotechnical modeling at the University of Bristol. Design details of the shear stack employed for the benchmark test are illustrated in Figure 2.2. The stack works in conjunction with the University of Bristol’s shaking table, a 3m by 3m platform driven by eight 70kN servo-hydraulic actuators to give full control of motion in all six degrees of freedom. Despite this capability, the shear stack is designed to be subjected solely to uniaxial dynamic excitation in the y-direction.

Dry Hostun S28 sand was employed. The average weight of sand deposited across the reported tests is 722kg (±1%) implying a void ratio of 0.76 (±2%). The spread results from difficulties in depositing and levelling the sand close to the model structure, accidental spilling of sand outside of the shear stack, and inaccuracies of the crane scale used to measure mass. The fundamental frequency and damping of the shear stack when filled to a depth of 798mm and excited with a random (white noise) waveform at a strain level of 0.003% in the x-direction was recorded as 26.5Hz and 8% respectively. The depth of sand used during the test was 750mm.

The dimensions of the single degree-of-freedom (1DoF) model of the Euroseistest building are detailed in Figure 4.57. The mass of the model is measured as 130kg, of which 80kg are attributable to the ‘foundation’ and the remaining 50kg to the ‘structure’. The direction of shaking is from left to right in the cross section detailed on the left of the figure. An array of steel columns are fixed to the foundation and pinned to the structure. The model was configured with twelve 5mm thick columns. The resonant frequency and damping of this model configuration measured by securing the model foundation to the shaking table is 22.5Hz and 2.4% respectively. The foundation was embedded by 100mm into the soil deposit. An image of the benchmark test taken prior to testing is displayed in Figure 4.58.

Prior to testing, a digital spectrum analyser (Advantest series R9211) was used to measure the resonant properties of the system. A random signal from the analyser’s inbuilt signal generator was used to excite the shaking table along its y-axis with RMS acceleration of around 0.03g. Frequency response functions (FRFs) were calculated by connecting the analyser’s input channel
to a sensor measuring the \( y \) acceleration of the shaking table and the analyser’s response channel to a sensor measuring the \( y \) acceleration of the superstructure. Resonant frequencies are obtained from the measured FRFs and the analyser’s curve fitting algorithm is used to derive the corresponding complex pole value from which the modal damping was obtained. The measured FRF is presented in Figure 4.59 wherein resonant frequency and modal damping values are also inscribed. The 22.5Hz fixed-base response of the model structure is not apparent in the FRF. Instead a peak at 12.6Hz is apparent which represents the resonant frequency of the SSI system.

The shear stack is excited in the \( y \)-direction by two earthquake time histories: the Y-Y component of Friuli-San Rocco earthquake (15/09/76) and the N-S component of the Vrancea earthquake (04/04/77). In order to excite both the model (which resonates at 22.5Hz) and the soil (which resonates at small strain magnitudes at 26.5Hz) time was scaled by a factor of 5.4 \( (1/\sqrt{l}) \) where \( l \) is the scale factor for length). The 16.75 second Friuli-San Rocco earthquake was replayed over 3.1 seconds; the 40 second Vrancea earthquake, over 7.4 seconds.

![Image of the benchmark model structure](image)

**Figure 4.57. The geometry of the benchmark model structure.**
Figure 4.58. The benchmark test.

Figure 4.59. Frequency response function measured across the benchmark SSI system.
4.5.2 Linear substructuring of the benchmark SSI test

The benchmark shaking table test described in §4.5.1 will provide a source of experimental data relating to an SSI system that can be used to validate the substructuring test methodology. Linear modelling has previously provided a reasonable correlation with the benchmark test results (Pitilakis et al (2008)). Due to the elapsed time since the benchmark test was conducted, the physical substructure was no longer available for use in the substructuring SSI tests. Hence, the ‘physical’ and numerical parts of this system are both modelled numerically. Note, however, that in the following discussion the terminology and the subscripts $M$ and $P$ are retained in order to distinguish between the different components of the system.

For simplicity, a lumped mass model is adopted for both soil and structure. A schematic of the substructuring system used in the validation tests is that presented in Figure 4.11. The superstructure of the benchmark test is represented by a single degree of freedom oscillator with the following parameters: $m_P = 50\text{kg}$, $\omega_P = 22.5\text{Hz}$, $\xi_P = 5.8\%$. The frequency and damping of the superstructure are those that were recorded for the benchmark test structure in the ‘fixed-base’ condition (i.e. by securing the foundation of the structure directly to the platform of the shaking table). The soil-foundation system is similarly represented as a single degree of freedom oscillator but with parameters: $m_N = 120.5\text{kg}$, $\omega_N = 15.96\text{Hz}$, $\xi_N = 14.1\%$. This parameter set was derived following a frequency domain analysis and with reference to Figure 4.59.

The frequencies exhibited by the benchmark test exceed the current substructuring capability. Some time scaling is consequently required. A factor of four is chosen which reduces the resonant frequencies of the physical and numerical substructures to 5.6Hz and 4.0Hz respectively, and extends the duration of the seismic excitation signals accordingly.

The response of the system is assessed in terms of the acceleration measured on the transfer system ($a_{IP}$) which, in the benchmark test, corresponds to the horizontal accelerations recorded on the foundation of the 1DOF oscillator ($a_{fnd}$). A comparison between the recorded acceleration time histories is presented at benchmark-test scale in Figure 4.60(a). The SSI substructuring test appears to perform reasonably well for the Vrancea excitation but poorly for the Friulli San Rocco motion. The power spectra of the acceleration time histories presented in Figure 4.60(b)
unify the picture and reveals that for both excitation motions the substructuring test performs adequately up to 10Hz benchmark scale.

It has previously been demonstrated that FSCS shaking table substructuring has an applicable bandwidth of 10Hz which corresponds to 40Hz benchmark scale. The reason for the premature malfunction of the substructuring test is thought to be associated with two phenomena: firstly, the increasing nonlinearity of the soil-foundation system as the resonant frequency (measured at 12.6Hz in Figure 4.59) is approached; second, dynamic behaviour that is not taken into account by the substructuring test but is exhibited during the benchmark test. Figure 4.61 shows the rocking acceleration measured on the benchmark test foundation in both the time and frequency domain. Unlike the Vrancea input motion, the Friulli San Rocco excitation motion induces significant rocking of the benchmark test structure. Furthermore, the frequency band in which the rocking occurs coincides with the frequency band where the substructured and benchmark test data diverge. A better correspondence between benchmark and substructuring data will be achieved by addressing these issues either through the application of a nonlinear numerical substructure and by the development of the multi-axis shaking table substructuring method.

Figure 4.60. Acceleration (a) time history and (b) power spectra response to the (i) Vrancea and (ii) Friulli San Rocco excitation motions.
4.5.3 SSI substructuring using a nonlinear macroelement

The dynamic response of real soil within a SSI system subjected to seismic excitation is determined by three main non-linear phenomena: firstly, the irreversible elastoplastic constitutive response of the soil material itself, second, the unilateral soil–foundation interface deformations which may lead to uplift and, third, sliding of structures. To assess the effect that such non-linearities have on the stability and accuracy of the shaking-table substructuring methodology developed herein a non-linear soil model is to be implemented as the numerical substructure. The macroelement for shallow foundations, hereafter referred to as ‘the macroelement’, presented by Chatzigogos et al (2011) was selected for implementation due to its breadth of scope – it encompasses a wide range of combinations of soil and foundation-soil interface conditions that are of practical interest – and its efficiency.

The formulation of the macroelement is presented elsewhere (SERIES Deliverable D14.3) and will not be discussed here. The implementation of the macroelement within the substructuring system is illustrated schematically in Figure 4.62. In essence the macroelement acts as a non-linear stiffness term within the integration scheme of the numerical substructure. The static impedance of the macroelement is an experimental variable that will be referred to using the notation \( \omega_N \) – note that \( \omega_N \) relates to only the initial stiffness of the macroelement. \( \xi_N \) is set at 10%, a recommended minimum for numerical soil modelling. Three configurations of the
macroelement are used. The first provides a linear elastic response. The second provides significant non-linear soil hysteresis. The third increases the susceptibility of the physical substructure to sliding. All tests were conducted using 3\textsuperscript{rd} order FSCS outer loop control. The displacement record of the El Centro earthquake (NS component of the 1940 El Centro, Station: 117 El Centro Array #9, Component 270\degree) was chosen as the reference excitation signal.

\begin{figure}[h]
\begin{center}
\includegraphics[width=\textwidth]{macroelement_model.png}
\end{center}
\caption{Implementation of the macroelement model within the substructuring system.}
\end{figure}

### 4.5.3.1 Stability

The stability of the macroelement substructuring system was assessed by conducting tests using two different single DOF physical substructures, the first configured so that \( \omega_p = 1.9\text{Hz} \) and \( \xi_p = 2\% \), the second \( \omega_p = 4.8\text{Hz} \) and \( \xi_p = 2\% \); corresponding results are presented in Figure 4.63 and Figure 4.64, respectively. As before, tests were conducted by incrementing upwards the mass ratio \( \sigma \) until instability occurred. The last stable and the first unstable points are those that are presented. Some now familiar trends are apparent – there is a slight tendency for the stability to decrease with increasing both \( \omega_N \) and \( \omega_p \) – it is clear that the configuration of the macroelement has little influence of the stability of the system. As in §4.4.4, the precise configuration of the numerical substructure does little to affect the stability of the substructuring system.
4.5.3.2 Transfer system accuracy

The accuracy for the linear, hysteretic and sliding macroelement is addressed in terms of transfer system synchronisation in Figure 4.65, Figure 4.66, and Figure 4.67 respectively. Herein a constant mass ratio $\sigma$ of 0.17 has been adopted; $\omega_p$ is 4.83Hz; $\omega_N$ is incremented from 1Hz to 9Hz at 2Hz intervals; $\zeta_p = 2\%$ and $\zeta_N = 10\%$.

Three representations of accuracy are employed. The first, in part (a) of the figures, compares the desired displacement time history (i.e. the numerically calculated resultant displacement of the numerical substructure) with the achieved displacement time history (i.e. the experimentally
measured displacement of the shaking table). Such plots only reveal inaccuracies when errors are a significant proportion of time history magnitudes. Here, the desired and achieved time histories lie on top of one another and are seen as a single line.

Subspace plots of desired displacement against achieved displacement are presented in part (b) of the figures. Perfect synchronisation is represented by a straight diagonal line at 45°. Experimental data are not seen to deviate significantly from the ideal response. However, subspace plots do not reveal the high frequency, low magnitude response because the axis scaling is governed by the high magnitude, low frequency response.

Part (c) of the figures presents the synchronisation error (i.e. \( U_a - U_d \)) time histories, the most punishing evaluation of transfer system accuracy that reveals both low and high frequency inaccuracies. For the sliding macroelement with \( \omega_N = 3 \text{Hz} \) an uncharacteristically large error of 3.2% is recorded. Data analysis for this test reveals that the error was caused by a premature instigation of excitation – a shaking-table-operator error, not a transfer system synchronisation error. Discounting this value, peak errors for all tests are less than 1.9%. Additionally, the errors associated with the nonlinear macroelements are equivalent to those of the linear system. Numerical substructure non-linearity does not upset transfer system accuracy.
Figure 4.65. Substructuring accuracy of the linear macroelement assessed in terms of (a) desired ($U_d$) and achieved ($U_a$) displacement time histories, (b) synchronization, and (c) error ($U_a - U_d$) with $\omega_r = 4.83$Hz and $\omega_N$ magnitudes of (i) 1Hz, (ii) 3Hz, (iii) 5Hz, (iv) 7Hz, and (v) 9Hz.
Figure 4.66. Substructuring accuracy of the hysteretic macroelement assessed in terms of (a) desired \( (U_d) \) and achieved \( (U_a) \) displacement time histories, (b) synchronization, and (c) error \( (U_a - U_d) \) with \( \omega_P = 4.83 \text{Hz} \) and \( \omega_N \) magnitudes of (i) 1Hz, (ii) 3Hz, (iii) 5Hz, (iv) 7Hz, and (v) 9Hz.
Figure 4.67. Substructuring accuracy of the sliding macroelement assessed in terms of (a) desired ($U_d$) and achieved ($U_a$) displacement time histories, (b) synchronization, and (c) error ($U_a - U_d$) with $\omega_r = 4.83\text{Hz}$ and $\omega_N$ magnitudes of (i) 1Hz, (ii) 3Hz, (iii) 5Hz, (iv) 7Hz, and (v) 9Hz.
4.5.3.3 Typical results

Below, some typical results of the macrolelements substructuring system are introduced. All data are obtained with mass ratio $\sigma = 0.17$, $\xi_P = 2\%$ and $\xi_N = 10\%$. The response of the linear $\omega_P = 4.83$Hz system is assessed in terms of transfer functions in Figure 4.68. Two transfer functions are considered. The first (black lines) takes the bedrock acceleration (i.e. that applied to the macroelement) and the superstructure acceleration (i.e. that measured experimentally on the physical substructure) as input and output respectively. The second (grey lines) takes the foundation acceleration (i.e. that measured on the shaking table) and the superstructure acceleration as input and output respectively. As one moves from left to right across the figure the stiffness of the macroelement increases allowing more energy to be transmitted to the physical substructure thereby increasing the response. Some clear interaction phenomena are observed in that the resonant peaks of the substructured system do not align with the resonant peaks of its constituent parts. This is perhaps most evident when there is minimum separation between the resonant frequencies of the physical and numerical substructures – at $\omega_N = 5$Hz – and the physical substructure resembles a tuned mass damper.

Despite the conceptual flaws of applying a linear system analysis to a nonlinear system, transfer functions for the hysteretic and sliding macroelements are presented in Figure 4.69 and Figure 4.70 respectively. In these figures, the linear system response of Figure 4.68 is also presented by way of contrast. The transfer functions of the soft macroelements deviate most from the linear response. The reason being that the softest macroelements respond most to the low-frequency, narrow bandwidth of the excitation signal (pictured in Figure 4.71).

The failure of the excitation to induce any significant nonlinearity for stiffer macroelements is confirmed in the hysteresis loops of Figure 4.73 (hysteretic macroelement) and Figure 4.74 (sliding macroelement). (Note that the equivalent data for the linear macroelement is presented in Figure 4.72 for completeness.) Only at low stiffness do the loops broaden to produce significant hysteresis. The hysteresis associated with the 1Hz sliding macroelement is so severe that the physical substructure has in essence a base isolator which prevents the excitation from reaching the physical substructure. As a result the system exhibits significant attenuation as seen in Figure 4.70(a).
The response of the macroelement to varying magnitudes of excitation is assessed using a $\omega_N = \omega_P = 1.9$Hz test configuration. The upper half of the figure considers the hysteretic macroelement while the bottom half of the figure considers the sliding macroelement. As the excitation increases the characteristics of the hysteresis loops change and they become broader. As expected, forces and displacements increase with excitation magnitude for the hysteretic macroelement whereas only the displacements grow with the sliding macroelement.

Figure 4.68. Bedrock-superstructure (black lines) and foundation-superstructure (grey lines) acceleration transfer functions recorded for the linear macroelement with $\omega_P = 4.83$Hz and (a) $\omega_N = 1$Hz, (b) $\omega_N = 3$Hz, (c) $\omega_N = 5$Hz, (d) $\omega_N = 7$Hz, and (e) $\omega_N = 9$Hz (dashed lines).

Figure 4.69. Bedrock-superstructure acceleration transfer functions recorded for the hysteretic (black) and linear (grey) macroelements with $\omega_P = 4.83$Hz (solid line) and (a) $\omega_N = 1$Hz, (b) $\omega_N = 3$Hz, (c) $\omega_N = 5$Hz, (d) $\omega_N = 7$Hz, and (e) $\omega_N = 9$Hz (dashed lines).
Figure 4.70. Bedrock-superstructure (black lines) and foundation-superstructure (grey lines) acceleration transfer functions recorded when $\omega_P = 4.83\text{Hz}$ and (a) $\omega_N = 1\text{Hz}$, (b) $\omega_N = 3\text{Hz}$, (c) $\omega_N = 5\text{Hz}$, (d) $\omega_N = 7\text{Hz}$, and (e) $\omega_N = 9\text{Hz}$ (dashed lines).

Figure 4.71. Displacement and acceleration bandwidth of the seismic excitation.
Figure 4.72. Response of the linear macroelement when $\omega_p = 4.83$Hz and (a) $\omega_N = 1$Hz, (b) $\omega_N = 3$Hz, (c) $\omega_N = 5$Hz, (d) $\omega_N = 7$Hz, and (e) $\omega_N = 9$Hz.

Figure 4.73. Response of the hysteretic macroelement when $\omega_p = 4.83$Hz and (a) $\omega_N = 1$Hz, (b) $\omega_N = 3$Hz, (c) $\omega_N = 5$Hz, (d) $\omega_N = 7$Hz, and (e) $\omega_N = 9$Hz.

Figure 4.74. Response of the sliding macroelement when $\omega_p = 4.83$Hz and (a) $\omega_N = 1$Hz, (b) $\omega_N = 3$Hz, (c) $\omega_N = 5$Hz, (d) $\omega_N = 7$Hz, and (e) $\omega_N = 9$Hz.

The residual displacements that commonly feature in nonlinear tests are apparent within the hysteretic and sliding macroelement substructuring systems. The time history responses of the different substructuring systems are compared in Figure 4.76. The left hand side of the figure presents the first 37 seconds of the strong motion excitation; the right hand side of the figure presents a detail view of the first large pulses of the excitation waveform. By the end of the test,
the residual displacements across the hysteretic and sliding macroelements equal 3.0mm and 12.2mm respectively resulting in residual offsets of the shaking table of 3.1mm and 12.3mm.

The effect of the nonlinearity on the response of the physical substructure is apparent in Figure 4.76(b)(i). Herein, the displacements (and by linear association forces) are generally smaller than those of the linear system. For this system, the stiffness degradation of the macroelement works to move the resonance of the numerical substructure away from that of the physical substructure. Hence macroelement nonlinearity produces a beneficial SSI effect.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{hysteresis_loops.png}
\caption{Hysteresis loops for the (i) hysteretic and (ii) sliding macroelement excited at (a) 1.2m/s$^2$, (b) 2.4m/s$^2$ and (c) 3.5m/s$^2$.}
\end{figure}
Figure 4.76. (a) Expanded and (b) detailed time histories of the $\omega_P = 4.83$Hz and $\omega_N = 3$Hz system: (i) relative displacement between the physical substructure and the macroelement, (ii) absolute displacement of the shaking table, (iii) relative displacement across the macroelement, (iv) excitation acceleration.
4.6 APPLICATION OF SECOND VARIANT OF FHT TECHNIQUE FOR SSI

The response of soils when subjected to dynamic loading is anisotropic and non-linear at strain levels well below its failure condition. The stiffness of the material is stress-level and strain-level dependant and its behaviour may change during an earthquake, as primary loading/unloading evolves to reloading and volumetric deformations occur. As a result soil is a complicated material to model numerically. This section takes these considerations into account and considers an inverted Soil Structure Interaction (SSI) test methodology wherein the physical substructure is the soil – tested in a soil container on the shaking table – and the numerical substructure is the structure. Such testing involves mounting the substructuring transfer system (i.e. an actuator or system of actuators) on the shaking table above a soil container filled with the test soil. The use of an actuator within a substructuring control loop in conjunction with a shaking table pushes the methodology into the category of hybrid testing.

Hybrid testing combines shaking tables and reactive systems (actuators, reaction walls, strong floors, etc) with substructures and real-time processing hardware for full system simulation (Reinhorn and Shao 2004). Unlike substructure testing conducted away from a shaking table, hybrid testing imposes the true inertial forces in the specimens since the dynamic excitation is applied directly to the physical substructure. The physical and numerical substructures interact through the application of interface equilibrium and compatibility conditions. This is achieved by measuring the displacement imposed on the numerical substructure at the numerical-physical interface and imposing this displacement on the numerical substructure, hence satisfying the equilibrium condition. Then the resultant interface force computed from the numerical model is then imposed on the physical system, satisfying the compatibility condition. The transfer system (consisting of a hydraulic or electrical actuator, for example) is required to impose the interface force calculated from the numerical substructure on the physical substructure. Compared to the shaking table substructuring described previously, the loop is reversed and the transfer system is operated in force not displacement control.

To investigate the performance of a hybrid test SSI system, a complete model of a hybrid test system is developed based on the hydraulic system modelling procedures detailed in earlier sections. The hybrid test model is described and analysed below.
4.6.1 Hybrid test modelling

The hybrid test system that will be modelled is the two DOFs system shown in Figure 4.77. Figure 4.77(a) presents the equivalent ‘entire’ test system, Figure 4.77(b) the hybrid model. The uppermost degree of freedom is modeled numerically in order to calculate the numerical reaction force \((f_{IN})\) at the interface between the numerical and physical substructure. The physical components of the hybrid test system include the soil (here the lowermost degree of freedom), the shaking table and the actuator which imposes the interface force. Subjected to excitation, the physical substructure responds and the displacement measured. This displacement is fed to the numerical substructure, producing a reaction force that is applied via the actuator. The shaking table applies the reference excitation \((r)\) to the physical substructure, the actuator applies the reaction force to the physical substructure.

\[
\begin{align*}
G_{Pr} &= \frac{y_P}{r_{st}} = \frac{(c_p s + k_p)}{m_p s^2 + c_p s + k_p} \quad (4.72) \\
G_{Py} &= \frac{y_P}{f_{IP}} = \frac{1}{m_p s^2 + c_p s + k_p} \quad (4.73)
\end{align*}
\]

Figure 4.77. Principle of a hybrid substructure testing.
Shaking table test techniques and fault rupture box testing for SSI

\[ G_{Nf} = \frac{f_{BN}}{y_f} = \frac{m_N (c_N s + k_N) s^2}{m_N s^2 + c_N s + k_N} \]  

(4.74)

Here \( r_{st} \) is the reference excitation resulting from shaking table; \( y_f \) is the interface displacement, the summation of the displacement of physical substructure resulting from reference excitation (\( y_{fr} \)) and that resulting from the numerical reaction force (\( y_{fr} \)); \( f_{IP} \) is the reaction force at the interface imposed by actuators with the desired value \( f_{BN} \); \( m_N, c_N, k_N \) are the mass, damping and stiffness of the numerical parts; and \( m_p, c_p, k_p \) are the mass, damping and stiffness of the physical parts.

Applying Newton’s second law to the forces on the piston, the resulting force equation of the actuator with the specimen is:

\[ A_p P_L = (m_p s^2 + c_p s + k_p) x \]  

(4.75)

Equation 4.75 can be used to obtain the transfer function of the actuator (\( G_{act} \)) from the command force to the applied force:

\[ G_{act} = \frac{K_p k_q (m_p s^2 + c_p s + k_p)}{\left[ (m_p s^2 + c_p s + k_p)\left( \frac{V}{4 \beta_c A^3} s + \frac{k_c}{A^3} \right) + s \right] K_p k_q} \]  

(4.76)

The nomenclature used within Equation 4.76 was introduced when addressing the modeling of shaking table substructure tests (i.e. when the soil was taken to be the numerical substructure). The control loop of the hybrid test system is illustrated in Figure 4.78, in which FSCS is used to compensate for the dynamics of both shaking table and actuators.

![Figure 4.78. Block diagram of a typical control strategy for a hybrid substructure testing.](image-url)
From Figure 4.78, the closed-loop transfer function of the hybrid test can be obtained:

\[
\frac{f_{IP}}{r} = \frac{G_{OFSCS} G_{Table} G_{Pr} G_{Nf} G_{act}}{1+G_{pf} G_{Nf} G_{act} G_{OFSCS}}
\] (4.77)

### 4.6.2 Analysis

A feasibility assessment of a proposed hybrid test must make reference to system stability. The closed-loop characteristic equation (CLCE) of the proposed system can be obtained from Equation 4.77:

\[
1+G_{pf} G_{Nf} G_{act} G_{OFSCS}
\] (4.78)

As known from previous sections,

\[
G_{act} G_{OFSCS} = G_{opt}
\] (4.79)

and the optimized transfer function of 3rd order open-loop FSCS is

\[
G_{opt} = \frac{\omega_{opt}^3}{(1+\omega_{opt})^3} \left( \omega_{opt} = 150\text{Hz} \right)
\] (4.80)

Substituting Equation 4.73, Equation 4.74, Equation 4.80 into Equation 4.78, the CLCE with variable parameter mass ratio (\(\sigma\)) is obtained:

\[
1+\sigma \left[ \frac{\omega_{opt}^3 \left( 2\xi_p s + \omega_p^2 \right) s^2}{(1+\omega_{opt})^3 \left( s^2 + 2\xi_N s + \omega_N^2 \right) \left( m_p s^2 + 2\xi_p s + \omega_p^2 \right)} - 1 \right] = 0
\] (4.81)

Where \(\omega_N\) and \(\omega_p\) are the natural frequencies and \(\xi_N\) and \(\xi_p\) are the damping ratios for the numerical and physical substructures respectively. The stability boundaries of the system can then be analysed using the root locus method developed in previous sections.
4.6.3 Hybrid test simulation of the benchmark test

The stability of the benchmark test described in Section 4.5 is presented in Figure 4.79. Herein, the physical substructure is the soil, giving as seen previously \( \omega_p = 15.96 \text{Hz} \) and \( \xi_p = 14\% \); the numerical substructure is the benchmark test oscillator giving \( \xi_N = 5\% \). For the analysis the numerical substructure frequency is varied between 1 to 50Hz while the actual value used in the benchmark test was 22.5Hz. The output of the analysis is presented in Figure 4.79 in which the benchmark test stability location is marked and found to be stable. Hence simulated hybrid test data can be obtained and compared to those equivalent data obtained using alternative test methodologies.

![Figure 4.79. Hybrid testing of the Benchmark.](image)

4.6.4 Comparison between hybrid testing and real time dynamic substructuring

Here, the two variants of substructuring are contrasted, the first variant being real time dynamic substructuring (where the numerical model comprises the soil and the physical substructure comprises the structure), and the second hybrid testing (where the numerical model comprises the structure and the physical substructure the soil). The simulated displacement of the hybrid test structure (i.e. the numerical substructure) is compared with that recorded for the entire system experimentally (i.e. during the benchmark test) and that recorded during the shaking table
substructuring test (i.e. the physical substructure) in Figure 4.80. The response is that induced by the N-S component of the Vrancea earthquake (04/04/1977). The agreement between the different test methodologies is reasonable, validating the use of the hybrid test method and the shaking-table substructuring test method for SSI studies on the shaking table.

![Graph showing structural displacement recorded using three alternative test methods.](image)

**Figure 4.80.** Structural displacement recorded using three alternative test methods.

### 4.6.5 Discussion

The hybrid test method for linearised SSI studies has herein been validated through simulation. However, there are a number of aspects of the method that require further attention before the method can be used in practice.

As with all shaking table testing, the size of the specimen (the physical substructure) is limited by the capacity of the earthquake simulator. Ideally, for securing fast, accurate control of
actuation, the capacity of the shaking table should be far in excess of model mass. However, as discussed in Section 2, shaking table tests of geotechnical models generally involve large volumes of soil in an attempt to reduce the effects of the specimen boundaries. Filled with dry sand, the large and small shear stacks at the University of Bristol are 200% and 33% of the mass of the shaking table platform. It is reasonable to expect that such levels of mass will have significant effects on the performance of the shaking table, modifying the dynamic characteristics. The effects of specimen-table interaction would need to be compensated for in order to guarantee the stability and accuracy of a hybrid test.

Furthermore, thought needs to be given to the practicalities of mounting servo hydraulic actuators above soil containers on shaking tables. Seismically rigid reaction frames to support the actuator are required to prevent feedback signals becoming corrupted with components associated with reaction frame resonance. It has been demonstrated previously that high frequency components have a detrimental effect on the performance of FSCS.

Finally, as mentioned in the introduction to this section, soil is a highly nonlinear material whose stiffness and damping characteristics are time variant. It has been shown previously that the properties of a linear physical substructure have a dominant influence on the stability of a substructuring test system. By way of illustration, the $\xi_p = 10\%$ and $\xi_p = 30\%$ stability boundaries have been added to Figure 4.79. These represent rough limits to the damping of typical SSI systems. Such variations of damping are seen to bring about instability. The sensitivity of the stability of the linearised hybrid system to the resonant frequency of the soil is addressed in Figure 4.81 for a $\omega_N = 2$Hz, $\xi_N = 5\%$ numerical substructure. A near identical set of curves result for $\omega_N = 5$Hz, 10Hz and 20Hz – the characteristics of the numerical substructure are once again seen to have little impact on stability. It is the dynamic characteristics of the physical substructure (i.e. the soil) that plays the dominant role in determining the system stability. Thus, before hybrid testing of authentic SSI systems can occur in practice, the stability analysis method for nonlinear hybrid test substructures needs to be developed.
Figure 4.81. The sensitivity of the hybrid test stability to physical substructure parameters.
5 INVESTIGATION OF FAULT RUPTURE PROPAGATION

5.1 INTRODUCTION

As part of Phase VI of the JRA 3.4, the Laboratory of Soil Mechanics (LSM) of NTUA has conducted a series of 1g experiments in the Fault Rupture Box (FRB), to investigate fault rupture propagation through soil and its interaction with embedded caisson foundations. Besides from being typical for bridge structures, the choice of embedded caisson foundations offers the possibility of observing strongly nonlinear phenomena, such as the diversion and bifurcation of the fault rupture path.

A series of 1g FRB tests were conducted, investigating: (a) the style of dip-slip faulting (normal and reverse), and (b) the position of the foundation relative to the fault rupture. The "bedrock" is subjected to movement due to fault rupture of vertical offset $h$ at a dip angle of 45°. To investigate the effect of the location of the structure relative to the fault rupture, the foundation is placed parametrically at distance $s$ from the outcropping location of the unperturbed (i.e. free field) fault rupture. The displacements of the foundation, $\Delta x$, $\Delta z$, and the rotation $\theta$, as well as the deformation of the soil mass were recorded during the experiments through image analysis and a laser scanning of the soil surface. In the first case, digital high-resolution cameras are utilized to capture photographs during the incrementally imposed fault rupture displacement $h$, which are then processed through image analysis. In the latter case, a novel technique was developed. After each displacement increment, the model surface was scanned with 8 laser displacement transducers, which travel along the specimen at a constant speed, producing a digital survey of the deformed surface.
The embedded foundation acts as a kinematic constraint, altering substantially the path of the fault rupture. The response of the foundation strongly depends on the position of the caisson relatively to the rupture. The tests conducted are in very good agreement with similar centrifuge model tests.

Moreover, an attempt was made to numerically simulate the conducted experiments. Analyses using finite element models were carried out and the results are quite encouraging.

Finally, a methodology has been proposed for the estimation of the structural stressing due to faulting.

5.2 DESCRIPTION OF THE LABORATORY FACILITIES AND EQUIPMENT

5.2.1 NTUA/LSM Fault-Rupture Box

Designed and constructed in-house, the Fault-Rupture Box (FRB) is designed to simulate quasi-static Fault Rupture Propagation and Fault Rupture – Soil – Foundation – Structure Interaction.

The apparatus (Fig. 5.1a and 5.1b) is essentially a split container (i.e. a glass walled box with a split base) of internal dimensions 2.60 x 1.10 x 0.90 m (length x height x width) having the capability of testing soil specimens up to 1 m deep. More specifically, the FRB consists of two parts: the fixed part and the movable part. The desired displacement is imposed on the movable part which moves downwards or upwards for the simulation of the normal or reverse fault rupture respectively. In general, the angle of the rupture angle is adjustable from 45° to 90°. At the two larger sides of the box there have been placed composite transparent barriers made of polycarbonate sheet at the outside, for rigidity, durability and safety, and of glass at the inside, in order to achieve minimum friction during the simulation of the rupture and simultaneously prevent the scratching of the soft outer wall surface by the sand grains. Utilising these composite barriers both the observation and the monitoring of the evolution of the fault rupture propagation, the deformation of the soil mass and the displacement of the structure that may exist are feasible.

The movement of the split base is realized by means of an electromechanical actuator (screw-jack) (Fig. 5.1d) with a capacity of moving 5 tones to a maximum displacement of 20 cm at a
controllable speed ranging from 0 to 5 cm/sec. The direction and the speed of the movement imposed by the actuator are controlled through an electrical controller (Fig. 5.1c).

Figure 5.1. a) The Fault-Rupture Box; b) configuration and dimensions of the apparatus; c) controller of the device; and d) electromechanical actuator (screw-jack).
5.2.2 NTUA/LSM Sand Pluviation Device

Modelling of physical problems in the laboratory requires the reassurance of the following three conditions: accuracy, reproducibility and repeatability. In the case of geotechnical physical modelling, the primary step towards the achievement of these three conditions involves controlling the soil model properties. Considering dry, sandy soil samples, this reduces to the control of the soil relative density, provided that the material properties are known.

A sand raining device has been designed and is currently used in the Laboratory of Soil Mechanics in NTUA for the preparation of sandy soil models of controllable density, used in fault rupture and shaking table experiments. The raining system, shown in Figure 5.2, consists of a bucket hanged from a beam, which can travel with a controllable speed, thanks to an electric motor, along a pair of parallel beams fixed on the roof of the laboratory. Another motor makes the bucket move in the vertical direction, sliding along the two vertical support beams. The bucket movement in both directions is operated by an electronic system, which allows remotely controlling of the pluviation speed and height during the sample preparation.

After the bucket is filled with sand, it is fixed at a certain height above the model container (in our case over the fault rupture box [Fig. 5.2b]), the shutter is removed, and sand flows like a curtain through an elongated opening of controllable aperture. During pluviation the bucket is moving automatically back and forth covering the whole length of the soil container. Pluviation is conducted in steps of about 5 cm of soil depth. At each step the pluviation height (height of the bucket) is increased proportionally to achieve uniform distribution of the density with depth.

The achieved soil sample density during raining depends on a) the drop height and b) the discharge rate of the sand. The drop height is specified by the vertical position of the soil bucket whereas the sand discharge rate is controlled by the bucket opening and the velocity of its movement in the horizontal direction (pluviation speed). The pluviation procedure has been calibrated in the LSM for the “Longstone” sand which has been used for the experiments described herein, and there are specific combinations of a) the pluviation height, b) the pluviation speed, and c) the discharge aperture that produce soil specimens of various yet controllable relative densities (Fig. 5.2c).
Figure 5.2. (a) The NTUA/LSM sand pluviation device, (b) the pluviation device above the Fault Rupture Box, (c) sand pluviation device calibration results: soil relative density as a function of pluviation height, speed, and aperture size.
5.2.3 NTUA/LSM Monitoring and Recording Equipment

The instrumented observation – monitoring of the experiments takes place by the use of instruments outside of the model. More specifically:

- A digital camera is used to take pictures of the model from a fixed position outside the fault rupture box. Quite a few pictures per test are being taken at progressively increasing fault displacements. The photographic data are then analyzed using the Geo-PIV software, written by White et al. [2003], to calculate displacements and/or shear strains developed within the soil or at the structure.

- A moving row of 8 Waycon laser displacement transducers is set above the free surface of the model to monitor and record the progress of fault actuation during the test and to validate and complement the image analysis results (Fig. 5.3). The row of the laser transducers is placed perpendicularly to the axis of the model and vertically. The row is assigned on a device that can move horizontally from one end of the model to the other scanning, in this way, the surface of the model during each displacement increment. Due to this procedure it is possible to digitally reproduce the deformed relief after each increment of fault rupture dislocation $h$. This technique was used to validate the numerical models set to simulate the investigated problem as described later below.

- The data from all the instruments are gathered through proper cables and recorded in the Vibration Research 8500 data acquisition system of the Laboratory.

5.3 DESCRIPTION OF THE EXPERIMENT LAYOUT

5.3.1 Problem under consideration

We consider the problem graphically depicted in Figure 5.4. A massive caisson foundation of dimensions $10 \times 5 \times 5$ m ($H \times B \times D$) fully embedded in a stratum of dense sand of 15 m depth. The relative density of the soil stratum is approximately 80% and the weight of the caisson foundation is 20 MN.

The bedrock is subjected to movement due to $45^\circ$ dip-slip fault rupture (normal or reverse) with a vertical component $h$, while the caisson position $s$ relative to the fault (defined as the horizontal
distance between the caisson right corner and the free field rupture outcrop at the caisson base level) is parametrically investigated.

The displacements of the caisson, $\Delta x$, $\Delta z$, and $\theta$, and complementary the displacements of the soil mass are recorded during the experiment. The number of the totally examined different cases are 8, and are synopsized in Table 5.1.

### 5.3.2 Experimental Layout

Taking into account the capacity of the Fault Rupture Box (FRB) the chosen model scale is 1:20, which is considered appropriate for the 1g simulation of the prototype problem, while the choice of dimension and materials used in the model is based upon the rules of similarity (Gibson, 1997).

The out-of-plane rigidity of the FRB side walls provides a kind of symmetry plane, allowing the simulation of the problem using only half of the caisson out-of-plane width.

Due to the adequate width of the FRB, it is feasible to simultaneously accommodate two cases during one experiment: the model of the caisson (in fact of half out-of-plane width) is placed on the one transparent side of the device while the free field or a second caisson model (in a different position, however) is investigated on the other side. The relative distance between the two caissons is assumed adequate so as to avoid important interaction between them.

### 5.3.2.1 Simulation of the Caisson Foundation

The experimental model of the rigid caisson (Fig. 5.5) was made of steel of density 8 Mg/m$^3$. The choice of the material regarded as suitable to achieve simultaneously the desired similarity between model and prototype scale both in terms of dimensions (N) and in terms of weight (N$^3$).

The dimensions of the half model were 0.5 x 0.25 x 0.125 m ($H \times B \times D$). The caisson was carefully positioned vertically with its upper boundary set at the level of the soil surface.

The side of the caisson placed on the transparent wall of the FRB was covered with a graphical white noise pattern to assist the displacement estimation by the image analysis software. Four
discernible optical targets have been also installed to make easier a redundant manual image analysis.

### Table 5.1. Synopsis of the experimental series conducted in the Fault Rupture Box of the Laboratory of Soil Mechanics, NTUA.

<table>
<thead>
<tr>
<th>Normal Fault</th>
<th>Reverse Fault</th>
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<td>Free Field</td>
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<td>FRSF Interaction – $s / B = 0.16$</td>
<td>FRSF Interaction – $s / B = -0.96$</td>
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<td>FRSF Interaction – $s / B = 0.38$</td>
<td>FRSF Interaction – $s / B = -0.04$</td>
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<td>FRSF Interaction – $s / B = 0.80$</td>
<td>FRSF Interaction – $s / B = 0.66$</td>
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Figure 5.3. Moving row of laser displacement transducers for scanning and recording of the soil deposit surface deformation.

Figure 5.4. Schematic of the studied problem indicating the basic parameters and dimensions at prototype scale.
5.3.2.2 Simulation of the Soil Deposit

The backfill consisted of dry “Longstone” sand, a very fine and uniform quartz sand with $D_{50} = 0.15$ mm and uniformity coefficient $D_{60}/D_{10} = 1.42$, industrially produced with adequate quality control. The grain size distribution curve for the sand is shown in Figure 5.6. The void ratios at the loosest and densest state have been measured in the laboratory. Based on the procedure described by Kolbuszewski (1948) the following values hold: $e_{\text{max}} = 0.995$, $e_{\text{min}} = 0.614$, and $G_s = 2.64$.

Direct shear tests were carried out to define peak and post-peak strength characteristics of the sand. Tests were performed on medium loose $D_r = 45 \pm 0.02\%$ and dense specimens $D_r = 80 \pm 0.07\%$ and for a normal stress range from 13 kPa (due to the weight of the top cap only) to 300 kPa. The low normal stress is more representative of the stress level prevailing in the 1g tests.
Loose specimens were prepared by raining the sand into the box while dense specimens were obtained by tapping the box after raining. The loose specimens have shown critical state behavior. The angle of shearing resistance appears to depend strongly on stress level and for stresses higher than 120 kPa $\varphi' \approx 32^\circ$, while for stress levels lower than 100 kPa $\varphi'$ increases up to $47^\circ$ at normal stress $\sigma = 13$ kPa. For the dense specimens, the angle of shearing resistance increases to $\varphi' \approx 35^\circ$ for higher stress levels and to $51^\circ$ for the lowest normal stress. These values drop after displacement of 6 mm to post-peak values similar to the peak strength of the medium-loose specimens, indicating an angle of dilation $\psi \approx 6^\circ$.

The relative density of the soil deposit simulated in the described experiments herein was $D_r = 80\%$. Given that the scale used throughout the experimental sequence is 1:20 and the thickness of the soil deposit in prototype units is 15 m, the soil specimen prepared was 75 cm thick.

Figure 5.6. Grain size distribution curve of the used Longstone sand.
5.3.3 Model Preparation

The model preparation begins with the sand pluviation within the FRB using the LSM sand pluviation device.

To achieve the desired density during the pluviation procedure (in this experimental series 80%), every time the procedure started, the pluviation height, the pluviation velocity, and the aperture of the bucket were defined. The choice of the suitable values of these three parameters was in accordance with Figure 5.2c.

The layering of the sand takes place in layers of approximately 5 cm. At the end of each layer pluviation a line of colored sand was dumped behind the transparent walls of the FRB (Fig. 5.7) to create a pattern to assist the observation of the rupture propagation through the soil.

In the case of the free field simulation, the procedure is repeated until the total height of 75 cm.

In the case that a caisson is also placed in the model, the pluviation is repeated, as described above, for the first 25 cm of soil (5 m in prototype units). At this height the foundation base level is reached and the caisson model is placed at its designated position. The foundation is placed carefully, achieving minor initial displacements, to face the transparent wall of the box. Thereafter, the remaining 50 cm of soil are pluviated in the same way.

After the model preparation the instrumentation takes place: the laser displacement transducer row is installed above the free surface of the model and the digital cameras are positioned in a suitable position at each side. Prior to any “bedrock” displacement imposing, the model undeformed shape (i.e. the initial condition) is recorded to use as a reference for the estimation of the caused displacements during the fault rupture propagation.

The FRB moving base displacement is imposed in small steps / increments —each of the order of 2mm vertical displacement. Following every increment of the imposed displacement, the “current” condition of the model is recorded so that by the end of the procedure (i.e. after 20 cm of fault vertical dislocation [4 m in prototype scale]), the evolution of the whole phenomenon is recorded.

In the following section the results from the experimental series are presented and discussed.
Figure 5.7. Model preparation – sand layers of approximately 5.5 cm and colored sand lines.
5.4 EXPERIMENTAL RESULTS

The experimental series consists of 4 experiments, two of which simulate normal fault rupture and the rest reverse fault rupture. Given that the side walls of the FRB provide a symmetry plane and the width of the box is adequately large, as discussed above, two cases were being simulated during each experiment. In this way, a total of 8 cases were investigated and presented herein. The results of the experiments conducted are presented below in respect to the fault type (normal or reverse) and the caisson relative position $s$.

Units are always in prototype scale, unless otherwise noted.

5.4.1 Normal Faulting

5.4.1.1 Free Field (Normal Fault)

The results of the free field case for normal fault rupture are presented first. A selection of images captured during the experiment at different stages of fault propagation is presented in Figure 5.8. In these snapshots the evolution of the primary (red solid line) and secondary (yellow dashed line) ruptures are illustrated for certain values of the base dislocation $h$. The distinction of the two ruptures to primary and secondary lies on the tectonic deformation magnitude of each one and not at the time of emergence.

Just after 16 cm of fault dislocation at the base of FRB, a quite steep rupture zone is observed at the lower 3/4 of the soil deposit (Fig. 5.8b). A bifurcation of this rupture appears for an increased displacement $h$ (Fig. 5.8c) however the fault deformation is localized on this distinct plane that first outcrops and a quite sharp scarp is formatted on the soil surface at a horizontal distance of about 2.5 m from the fault initiation point. When the fault throw is $h = 0.4$ m an opposite secondary rupture emanates. As the displacement of the model base increases another reverse secondary rupture path appears. Nevertheless, the mechanism does not experience any change, except from the fact that the localization zones, both primary and secondary, become wider. Between the two shearing zones a wedge shaped soil mass is forced to subside due to loss of support. After the full formation of the primary and secondary paths, the subsidence is of the same order as the fault movement (Fig. 5.8f).
The primary fault rupture emerged on the surface at a distance of about 3 m from the base discontinuity having an almost constant dip angle along its path, more than 70° (by 25° steeper than the base rupture angle). The greater propagation angle observed during the experiment is in agreement with former experimental results (e.g. Cole and Lade, 1984; Bransby et al, 2008b; Loli et al, 2010) and field observations of normal fault rupture patterns (see Bray et al., 1994a). Normal faults tend to refract on the soil–bedrock interface and propagate to the surface at increased dip angles; nonetheless the writers believe that the experimental results have overestimated this phenomenon. The significant difference in angles may be attributed to the stress conditions that are not properly simulated due to the scale and the 1g gravity conditions.

Having recorded the trace of the normal fault rupture propagation through the soil deposit, in the following experiments the interaction of the rupture and the foundation is simulated. Three characteristic positions of the foundation relatively to the fault were examined \( \frac{s}{B} = 0.16, 0.38, 0.80 \) corresponding to three different and distinct failure mechanisms. The results from the three tests are presented herein in terms of a) snapshots depicting the evolution of the phenomena, b) vector fields and contours of the displacements throughout the soil-caisson model as calculated utilizing the PIV method, contours of shear strains in the vicinity of the foundation (again using the image analysis technique), and measurements of the translational and rotational displacements of the caisson foundation.

It has to be noticed that due to accidental movement of the camera during the simulation of this case it was not possible to extract any valid result using the image analysis method (PIV).
Figure 5.8. Snapshots of model displacements / deformations for characteristic values of fault movement $h$. Primary rupture is marked with red solid line, while secondary ruptures are denoted in yellow dashed lines [normal fault – free field].
5.4.1.2 Rupture – Caisson Interaction: \( \frac{s}{B} = 0.16 \) (Normal Fault)

In this test the caisson was positioned so that the free field rupture would cross its base in the vicinity of its right corner as shown in Figure 5.9a. Characteristics snapshots captured during the experiment are shown in Figure 5.9 demonstrating the evolution of the phenomenon. For \( h = 0.16 \) m, it is shown that significant deformation occurs on the hanging wall and underneath the right corner of the caisson, with evident localization almost parallel to the interface of the foundation with the surrounding soil. This is an indication that the already steep fault rupture plane becomes even steeper, almost vertical, due to the divergence of the fault path by the caisson. The consequent movement of the caisson to the right causes a field of active pressures behind the foundation (at the foot wall). Further increase of the base displacement up to 35 – 40 cm leads to the development of a distinguishable active failure prism (Fig. 5.9c & 5.9d). In this way, the failure of the soil behind the caisson facilitates the development of another rupture zone which becomes the primary shearing zone for \( h \) greater than 0.6 m (Fig. 5.9e & 5.9f). The displacements and strains of the soil – caisson system are presented in Figures 5.10 – 5.12 which verify the failure mechanisms described above. The response of the foundation due to faulting is illustrated in Figure 5.13. The caisson undergoes large displacements (horizontal and vertical) as well as rotation which reaches 14° for 2 m of base vertical movement. It is obvious that there is much stressing on the structure.
Figure 5.9. Snapshots of model displacements / deformations for characteristic values of fault movement $h$. Primary rupture is marked with red solid line, while secondary ruptures are denoted in yellow dashed lines [normal fault – caisson with $s/B = 0.16$].
Figure 5.10. Vector field of displacements on deformed shape for characteristic values of fault movement $h$. Foundation displacements are denoted in red [normal fault – caisson with s/B = 0.16].
Figure 5.11. Vertical displacement contours on undeformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [normal fault – caisson with $s/B = 0.16$].
Figure 5.12. Strain contours on deformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [normal fault – caisson with s/B = 0.16].
Figure 5.13. Evolution of the caisson displacements and rotation as a function of fault movement $h$. [normal fault – caisson with s/B = 0.16].
In this test the caisson was positioned a little bit further to the right so that the free field rupture would cross the caisson base close to its middle. The evolution of the rupture propagation and the response of the soil and the structure are presented in Figures 5.14 – 5.17. Similarly to the previous case, the applied tectonic displacement forces the caisson to rotate clockwise and consequently the active failure prism is developed again behind the foundation. The failure wedge is visible for h greater than 22 cm (Fig. 5.14b) while at the same time no rupture (either primary or secondary) can be discerned. However, the contours of displacements (Fig. 5.16) show that the fault rupture propagates along the right side of the caisson. Further increase in the base dislocation causes bifurcation and diffusion of the shearing at both sides of the foundation. For h = 0.38 m (Fig. 5.14c) the secondary rupture is formed at the right of the caisson. The primary rupture starts propagating with smaller angle towards the foot wall and for h = 0.50 m it just crosses the caisson at its left bottom corner (Fig. 5.14d). Then, the primary rupture assisted by the active failure wedge propagates up to the soil surface (Fig. 5.14e & 5.14f). As seen in Figure 5.18, the foundation is quite stressed in this case too. Nevertheless, the displacements are smaller in this position (s/B = 0.38) compared to the previous one (s/B = 0.16).
Normal Fault – Caisson Interaction
\( s / B = 0.38 \)
F2010CAI1-001-03052010

(a) \( h = 0 \) m  
(b) \( h = 0.22 \) m  
(c) \( h = 0.38 \) m  
(d) \( h = 0.50 \) m  
(e) \( h = 0.80 \) m  
(f) \( h = 1.22 \) m

Figure 5.14. Snapshots of model displacements / deformations for characteristic values of fault movement \( h \). Primary rupture is marked with red solid line, while secondary ruptures are denoted in yellow dashed lines [normal fault – caisson with \( s/B = 0.38 \)].
Figure 5.15. Vector field of displacements on deformed shape for characteristic values of fault movement $h$. Foundation displacements are denoted in red [normal fault – caisson with $s/B = 0.38$].
Figure 5.16. Vertical displacement contours on undeformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [normal fault – caisson with $s/B = 0.38$].
Figure 5.17. Strain contours on deformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [normal fault – caisson with $s/B = 0.38$].
Figure 5.18. Evolution of the caisson displacements and rotation as a function of fault movement $h$. [normal fault – caisson with $s/B = 0.38$].
5.4.1.4 Rupture – Caisson Interaction : \( s / B = 0.80 \) (Normal Fault)

Finally, the caisson was positioned further to the right so that the free field rupture would cross the foundation base close to its left end. The evolution of the fault-rupture-soil-foundation interaction mechanisms is presented in Figure 5.19 and verified in Figures 5.20 – 5.22. For \( h = 0.16 \) m the rupture seems to diverge to the left of the caisson leaving the foundation totally in the hanging wall (Fig. 5.19b). The rupture outcrops for \( h = 0.31 \) m (Fig. 5.19c). Contrary to the previous two cases, the response of the foundation to faulting is dominated by the vertical displacement of the foundation (Fig. 5.23) and quite interestingly a counterclockwise rotation of the caisson is now initiated. Consequently, active conditions are now developed at the left of the caisson (for \( h = 0.66 \) m) helping a secondary opposing rupture to form and propagate (Fig. 5.19d – 5.19f).
Figure 5.19. Snapshots of model displacements / deformations for characteristic values of fault movement $h$. Primary rupture is marked with red solid line, while secondary ruptures are denoted in yellow dashed lines [normal fault – caisson with $s/B = 0.80$].
Figure 5.20. Vector field of displacements on deformed shape for characteristic values of fault movement $h$. Foundation displacements are denoted in red [normal fault – caisson with $s/B = 0.80$].
Figure 5.21. Vertical displacement contours on undeformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [normal fault – caisson with $s/B = 0.80$].
Figure 5.22. Strain contours on deformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [normal fault – caisson with s/B = 0.80].
Figure 5.23. Evolution of the caisson displacements and rotation as a function of fault movement $h$. [normal fault – caisson with $s/B = 0.80$].
5.4.2 Reverse Faulting

5.4.2.1 Free Field (Reverse Fault)

The results of the free field case for reverse fault rupture are presented in Figures 5.24 – 5.26. In agreement with published experimental research results (Bransby et al., 2008a and b; Loli et al., 2010), the reverse fault ruptures outcrop at the free surface of the soil deposit for greater base displacement compared to the corresponding value of the normal fault case. The first rupture (yet the secondary one) appears for $h = 0.46$ m (Fig. 5.24b). Although there is no visible strain localization, a wide failure zone can be identified in the image, which starts from the fault application point and propagates towards the soil surface within approximately the 1/3 of the soil specimen. This is also indicated by the strain contours (Fig. 5.26a), which reveal a possible shear zone propagating towards the soil surface but not a distinguishable failure plane. After 0.7 m of additional fault throw ($h = 1.18$ m) a displacement discontinuity (scarp) appears on the surface at a horizontal distance of about 16 m from the fault initiation point and the shear failure zone is visible throughout the soil layer. The shear failure is again distributed to a zone rather than a distinct surface. On further fault displacement ($h > 2$ m) the shear failure zone is enveloped by the two failure planes (Fig. 5.24e & 5.24f) and the failure pattern does not change any more.
Figure 5.24. Snapshots of model displacements / deformations for characteristic values of fault movement $h$. Primary rupture is marked with red solid line, while secondary ruptures are denoted in yellow dashed lines [reverse fault – free field].
Figure 5.25. Vertical displacement contours on undeformed shape for characteristic values of fault movement $h$ [reverse fault – free field].
Figure 5.26. Strain contours on deformed shape for characteristic values of fault movement $h$ [reverse fault – free field].
5.4.2.2 Rupture – Caisson Interaction: \( s/B = -0.96 \) (Reverse Fault)

In this experiment a marginal situation is simulated: the foundation is positioned quite on the left of the base discontinuity so that the rupture crosses the caisson at its top right corner. Snapshots from the specific experiment are shown in Figure 5.27. The corresponding displacement and strain field vectors and contours are illustrated in Figures 5.28 – 5.30. A careful observation of these figures reveals a slight diversion of the rupture to the right of the caisson (towards the upward moving hanging wall). Therefore, the foundation stays almost completely on the foot wall without experiencing any noteworthy movement (Fig. 5.31). It is worthy to notice that the displacement of the caisson initially increases rather linearly with the applied model base movement for \( h < 0.4 \) m where the soil practically remains in the elastic regime (i.e. before the full development of a discernible shearing zone) and then, after the outcropping of the fault plane, the foundation stays almost still.
Normal Fault – Caisson Interaction

\( s / B = -0.96 \)

F2010CAI1-003-23072010

(a) \( h = 0 \) m

(b) \( h = 0.37 \) m

(c) \( h = 0.44 \) m

(d) \( h = 0.66 \) m

(e) \( h = 1.00 \) m

(f) \( h = 1.78 \) m

Figure 5.27. Snapshots of model displacements / deformations for characteristic values of fault movement \( h \). Primary rupture is marked with red solid line, while secondary ruptures are denoted in yellow dashed lines [reverse fault – caisson with \( s/B = -0.96 \)].
Figure 5.28. Vector field of displacements on deformed shape for characteristic values of fault movement $h$. Foundation displacements are denoted in red [reverse fault – caisson with $s/B = -0.96$].
Figure 5.29. Vertical displacement contours on undeformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [reverse fault – caisson with $s/B = -0.96$].
Figure 5.30. Strain contours on deformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [reverse fault – caisson with $s/B = -0.96$].
Figure 5.31. Evolution of the caisson displacements and rotation as a function of fault movement $h$. [reverse fault – caisson with $s/B = -0.96$].
5.4.2.3 Rupture – Caisson Interaction: \( s / B = -0.04 \) (Reverse Fault)

After that, the case of the “free-field” rupture trace hitting the bottom right corner of the foundation was simulated. The progress of the failure mechanisms is depicted in Figures 5.32 – 5.35. The stressing of the right part of the model (in front of the caisson) is observable only after applying more than 40 cm vertical displacement at the base (Fig. 5.32b). The rupture propagates to the surface with an angle substantially larger compared to the free field case indicating the significant interaction between the rupture and the caisson. Indeed, for \( h = 0.75 \) m, the rupture hits the right bottom corner of the caisson and deviates to the right (Fig. 5.32c). Then it goes up almost vertically along the right side of the foundation and a step is formed at the soil surface adjacent to the caisson for \( h > 1 \) m (Fig. 5.32d). At the same time, a secondary rupture zone is developed which propagates with a smaller angle towards the left side of the caisson. However, this zone does not go beyond the caisson foundation level, at least for the applied base dislocation \( (h < 1.8 \text{ m}) \). The foundation in this case undergoes major displacements and rotations as shown in Figure 5.36.
Figure 5.32. Snapshots of model displacements / deformations for characteristic values of fault movement $h$. Primary rupture is marked with red solid line, while secondary ruptures are denoted in yellow dashed lines [reverse fault – caisson with $s/B = -0.04$].
Figure 5.33. Vector field of displacements on deformed shape for characteristic values of fault movement $h$. Foundation displacements are denoted in red [reverse fault – caisson with s/B = -0.04].
Figure 5.34. Vertical displacement contours on undeformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [reverse fault – caisson with $s/B = -0.04$].
Figure 5.35. Strain contours on deformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [reverse fault – caisson with s/B = -0.04].
Figure 5.36. Evolution of the caisson displacements and rotation as a function of fault movement $h$. [reverse fault – caisson with s/B = -0.04].
5.4.2.4 Rupture – Caisson Interaction : $s / B = 0.66$ (Reverse Fault)

Finally, the caisson was positioned further to the right so that the free field rupture would cross the foundation base close to its middle. The evolution of the fault-rupture-soil-foundation interaction mechanisms is presented in Figure 5.37 and verified in Figures 5.38 – 5.40. Similarly to the previous case the rupture is bifurcated and the shear zone diffuses towards both sides of the caisson. The primary branch of the fault zone deviates to the left (to the foot wall) and only little deformation occurs along the right side of the massive foundation. The latter moves substantially (Fig. 5.41); yet the experienced rotations are reduced compared to the previous case.
Figure 5.37. Snapshots of model displacements / deformations for characteristic values of fault movement $h$. Primary rupture is marked with red solid line, while secondary ruptures are denoted in yellow dashed lines [reverse fault – caisson with $s/B = 0.66$].
Figure 5.38. Vector field of displacements on deformed shape for characteristic values of fault movement $h$. Foundation displacements are denoted in red [reverse fault – caisson with $s/B = 0.66$].
Figure 5.39. Vertical displacement contours on undeformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [reverse fault – caisson with $s/B = 0.66$].
Figure 5.40. Strain contours on deformed shape for characteristic values of fault movement $h$. Foundation position is denoted in red [reverse fault – caisson with $s/B = 0.66$].
Figure 5.41. Evolution of the caisson displacements and rotation as a function of fault movement $h$. [reverse fault – caisson with $s/B = 0.66$].
5.4.3 Synopsis – Conclusions

The propagation of normal and reverse fault rupture through a dense sand layer as well as the interaction of the rupture with the soil and a caisson foundation were investigated through experiments conducted in the Fault Rupture Box device of the Laboratory of Soil Mechanics (NTUA).

The caisson foundation dimensions were selected as typical values for a medium-sized bridge and the experimental investigation focused on the influence of the relative position of the foundation to the rupture trace. For each one of the two fault types (normal and reverse), three different positions of the caisson were examined and the observed interaction phenomena were described above.

It has been proven that the massive foundation acts as a kinematic constraint preventing the propagation of the rupture in a manner similar to the free field case. Thus, significantly different failure mechanisms are developed. The presence of the caisson in the vicinity of the shearing zone can force the rupture either a) to totally diverge towards the foot or the hanging wall, or b) to bifurcate and diffuse around the caisson.

The exact interaction mechanism and the corresponding response of the foundation proved to be radically affected from the position of the caisson relatively to the rupture surface. Nevertheless, this stiff and massive foundation has an apparent advantage compared to more flexible types of foundation (e.g. pile groups, spread footings) that cannot diverge the fault rupture outside the structure area.

5.5 NUMERICAL ANALYSIS OF THE FAULT RUPTURE – SOIL – FOUNDATION INTERACTION PROBLEM

Triggered by the observations of the experimental investigation and in attempt to propose a methodology for the estimation of the structures stressing due to faulting, the experiments described above were reproduced numerically using the finite element analysis method. The procedure followed and the analyses results are discussed below.
5.5.1 Description of the Numerical Simulation Procedure

Several researchers have utilized the finite element method, satisfactorily simulating the phenomenon of fault rupture propagation in the free field [e.g. Bray et al., 1994a; Anastasopoulos et al., 2007; Loukidis et al., 2009] as well as the fault rupture – soil – foundation interaction [Anastasopoulos et al., 2009; Loli et al, 2010]. Most of the research conducted so far has been using 2-D modeling, mostly because of time limitations and lack of computational power. Nowadays, it is possible to realistically simulate the response of the foundation and the surrounding soil, using 3-D modeling of the problem, which is considered to be more accurate.

In order to simulate numerically the experiment conducted in LSM and described above, the FE code ABAQUS is utilized. The FE model dimensions are chosen to be the same with the dimensions of the physical model at prototype scale, thus 52m in length, 15m in depth of the soil stratum and 12m in width.

It should be noted that as was the case with the physical model, only half of the model was simulated, taking advantage of the symmetry along the model long axis. Note that the geometry of the model fulfils the requirement of \( Z = 4 \ H \), which was suggested by Bray et al. (1994b) in order to avoid boundary effects. That is applied both for the longitudinal and the transverse direction, on the plan view. The FE analysis model is shown in Figure 5.42 and some important dimensions are also depicted.

The soil material is modeled numerically with 8-noded hexahedral continuum finite elements in an adequately refined mesh. The unit weight of the soil was set equal to a representative value for dense Longstone sand \( \gamma = 16 \text{ kN/m}^3 \). The elastoplastic behaviour of the soil was simulated according to the validated methodology of Anastasopoulos et al. (2007): an elastoplastic constitutive relationship with the Mohr Coulomb failure criterion and isotropic strain softening is used. The applied strain softening rule assumes that the friction and dilation angles are linear decreasing functions of the octahedral plastic strain, \( \gamma^p_{\text{oct}} \), until they reach their residual-critical state values. In this way, the inherited shortcoming of the finite element method in modeling the width of failure surfaces, which arises from inadequate mesh refinement, is overcome. The ability of the developed constitutive relation to adequately represent the stress – strain behavior of dry sands has been verified by the satisfactory agreement with published direct shear test data.
The soil behavior is approximated as being divided in the following four phases: (I) *Quasi-elastic behavior* up to the point that the yield strain ($\gamma_y$) is reached; (II) *Plastic behavior* from the point that the soil yields to the point that the peak strength is reached (at shear strain $\gamma_{peak}$); (III) *Softening behavior* from the peak point to the point that critical state is reached and the soil is sheared at constant volume (at shear strain $\gamma_{res}$); (IV) *Residual behavior* for shear strain values larger than $\gamma_{res}$.

Additionally, the subroutine used for the implementation of the nonlinear behavior described above was further modified in our case to take into account the octahedral stress level at *every* iteration, as the stress level is an important parameter that defines the soil behavior during the fault rupture propagation through the soil: the internal friction angle of the soil material decreases as the stress level increases. Thus, the modified subroutine is capable of estimating the friction angle and the dilation angle accurately in any point of the soil mass. Recall, that according to the theory, the difference between the peak and the residual friction angle should be equal to the dilation angle. From shear tests conducted it is found that for relative density of about 80% and for the stress range anticipated during the specific experiments the dilation angle could be considered nearly constant, approximately 13°.

As far as the foundation is concerned, it was simulated through 3-D 8-node continuum elements. The foundation material behaviour was assumed linearly elastic with typical stiffness properties for steel. The interface between soil and caisson is modelled using contact elements to allow sliding, uplifting and/or separation (loss of contact), obeying to a rigid–perfectly plastic relationship.
Figure 5.42. Finite Element model for the case with the caisson system: (a) 3-D view of the model; (b) side view of the model, indicating the boundary conditions (case of normal fault). With the red dashed line the plane of symmetry is indicated, while with the yellow the caisson.

5.5.2 Results of the Numerical Analyses and Comparison with the Experiments

The results of the analyses are given in terms of: a) deformations and strain localizations within the soil mass (i.e. failure mechanisms), b) surface displacement profiles and c) foundation displacements \( \Delta x, \Delta z, \theta \). Moreover, the results of the experiments and the numerical analyses are compared side by side, demonstrating the similarities and some diversions and proofing the effectiveness of the numerical method in capturing the different components of fault rupture.
propagation in the free field and in the fault rupture – soil – foundation interaction (FRSFI) problems. The data from the moving laser displacement transducer row regarding the deformation of the free surface of the model are now utilized to compare with the numerical analyses.

Besides the fault type (normal or reverse), the parameter examined is once more the relative position of the foundation to the fault trace at the free field. Therefore, it is considered essential to study the evolution of the phenomenon in the free field before proceeding to the study of the FRSFI mechanisms.

It should be noted that for all graphs presented, the horizontal position axis (x) is plotted in such way that zero point represents the position of the fault initiation at the “bedrock”.

### 5.5.2.1 Normal Faulting

#### 5.5.2.1.1 Free Field (Normal Fault)

Figure 5.43 presents the results of the numerical analysis in terms of mesh deformations and the associated shear strain localization taking place at different stages of fault displacement. The analysis is in good accordance with the FRB test as far as the surface movements (fault offset) and the evolution of the phenomenon is concerned. This is the case for both the primary and secondary localization planes. Even the width of the zone that increases with the increase of the fault throw agrees with the remarks from the experiment. The aforementioned remarks are supported by Figure 5.44, which illustrates the comparison of the experiment and analysis with reference to the vertical displacements at the surface.

#### 5.5.2.1.2 Rupture – Caisson Interaction : $s/B = 0.16$ (Normal Fault)

Indicative results of the numerical simulation of this case focusing on the failure mechanisms developed at different stages of faulting are presented in Figure 5.45. It demonstrates that the analysis is in sufficient agreement with the experiment and captures the development and the evolution of the previously described mechanisms. Although the slope failure formed behind the caisson during normal fault rupture does not allow tracking of the surface rupture path with great
accuracy, the figure clearly shows the similarity between analysis and experiment regarding the FRSFI mechanisms.

Comparing the side views shows that the numerical analysis captures the mobilization of the three failure mechanisms, which have been described above. The numerically predicted value of the active failure wedge angle is in accordance with the experiment, while the mechanism in the right side of the caisson is bit wider in the experiment. Nevertheless, general pattern of behavior is very similar. Moreover, both analysis and experiment show the formation of a gap (loss of contact between the soil and the caisson) on the foot wall. However, the extent of this gap is underestimated in the analysis. This is also shown by the displacement profiles along the model surface in Figure 5.46. It is indicated that the numerical analysis slightly underestimates the steepness of the surface discontinuity and the extent of the gap formatted at the soil – caisson interface for fault throws larger than 0.5 m.

Despite the aforementioned discrepancies, the numerical analysis captures the overall behavior of the fault – caisson interaction. This is demonstrated in Figure 5.47, which shows that the analytical results are in agreement with the experimental results in view of the caisson performance (rotation and translational displacements) for all stages of fault loading.

5.5.2.1.3 Rupture – Caisson Interaction : s / B = 0.38 (Normal Fault)

The aforementioned failure mechanisms in this case (i.e. the main fault rupture plane and the secondary localizations caused by the caisson rotation) are schematically indicated in Figure 5.48, which presents the results of the numerical analysis in terms of plastic strains developed within the soil mass at different levels of fault displacement. The figure demonstrates the qualitative agreement between analysis and experiment with respect to the FRSFI mechanisms. The main failure plane appears for fault throw $h = 0.4$ m, at the same level of fault throw compared to the experiment. Analysis and experiment go side by side for almost every step during the evolution of the phenomenon. In accordance with the test, there is a diffused shear deformation zone underneath the caisson base which takes the form of a more localized failure plane as it propagates from the left corner of caisson to the soil surface. The analysis agrees with the experiment on the amount of fault throw needed to propagate the localization to the surface. Moreover, the analysis captures the mobilization of the aforementioned secondary localizations
on the hanging wall due to the caisson rotation (shearing along the caisson right sidewall and passive type wedge on the top right corner).

The displacement profiles in Figure 5.49 show that as it was also the case previously, although there is a small difference in the location of the fault outcrop, the general mechanism of the fault diversion is similar. The main discrepancy between analysis and experiment however, lies on the extent of the gap developed on the top part of the caisson’s left sidewall. The length of the gap is underestimated by the analysis similarly to the previous test, while the gap width is overestimated. Nevertheless, this disagreement does not necessarily indicate a limitation of the numerical methodology and could be probably attributed to 1g conditions during the scaled test. Figure 5.50 demonstrates a reasonable agreement between analysis and experiment regarding the caisson displacements, for different values of fault throw.

5.5.2.1.4  Rupture – Caisson Interaction : $s/B = 0.80$ (Normal Fault)

The numerically computed failure mechanisms for different stages of fault loading in the case of $s/B = 0.80$ are illustrated in Figure 5.51. The analysis agrees with the experiment showing that the fault is diverted on the left bottom corner of the caisson and then propagates towards the surface in a steep gradient. The analysis also predicts the secondary rupturing mechanism. The argument that the analysis agrees reasonably well with the experiment in terms of the FRSFI mechanisms taking place is also supported by the comparison of the surface displacements in Figure 5.52. In this case, the analysis successfully predicts the deformation of the surface and the movement of the foundation.

Figure 5.53 displays the performance of the caisson with respect to the applied fault throw in terms of displacements and compares the numerical results against the experimental ones. First, it is important to notice that in this case the caisson experiences far smaller rotations compared to the two previous tests. This verifies the previously formulated argument regarding the limited interaction of the caisson with the fault planes. Although the numerical analysis agrees with the experiment in view of the horizontal and vertical displacements for all stages of faulting, this is not the case for the caisson’s rotational movement. Although the numerically derived relationship between fault throw and consequent caisson rotation follows the same trend with the
experimental curve, the numerical analysis gives significantly lower rotation values for higher fault throw levels \((h > 1 \text{ m})\).

### 5.5.2.2 Reverse Faulting

#### 5.5.2.2.1 Free Field (Reverse Fault)

Figure 5.54 illustrates the numerically derived free field response for the reverse fault case and the comparison to the experimental results. The response is shown in terms of mesh deformations and associated plastic strains. In accordance to the experiment, there is no plastic deformation reaching the soil surface for less than 0.5m of fault displacement. A rupture appears at the surface for \(h = 1.2 \text{ m}\) however a distinct failure plane is not formatted until the fault throw doubles \((h = 2.5 \text{ m})\). In this case the first and the second failure planes are not easily separated, but it is encouraging that the shear deformation zone is of the same order in width. Once more, the tendency of the failure plane to bend over the footwall is quite observable. While the numerical analysis agrees with the experiment regarding the general failure pattern and its evolution, it predicts that the fault outcrops further away to the left compared to what happened in the experiment (in the numerical analysis the fault emerges about 1.5 m left of the 1g test location).

The comparison between numerical analysis and experiment in terms of vertical displacements \((\Delta z)\) along the soil surface occurring at different levels of fault throw is illustrated in Figure 5.55. Although the predicted soil surface deformed shape is steeper than the reality, the position of maximum gradient (i.e. the position of maximum relative vertical displacement) is approximately the same. The discrepancies may be attributed a) to the insufficiently simulated stress conditions in the 1g experiments and b) to the unavoidable limitation of the numerical analysis in modeling accurately the post peak soil behavior —that is why the comparison is better for small fault displacements (see curves for \(h = 0.2 \text{ m}\)) when most of the soil deforms elastically.
5.5.2.2 Rupture – Caisson Interaction: \( s/B = -0.96 \) (Reverse Fault)

The numerical analysis results are shown for this case in Figures 5.56 – 5.58. Although there is a “delay” in the emergence of the localization on the surface (the localization appears on the right of the caisson for 0.7 m of fault dislocation in the experiment but not before the fault throw reaches approximately 1 m in the analysis), the numerical analysis seems to capture the overall fault rupture – caisson interaction behavior. Two observations are noteworthy: a) the fault rupture deviates by almost 5 m compared to the free field trace due to the presence of the massive structure which b) stays firm at its initial position.

Figure 5.57 compares the numerical analysis with the experiment in terms of the vertical displacements occurring along the model surface for different values of fault dislocation. The agreement between numerical analysis and experiment is quite satisfactory for all fault throw levels. The only divergence between numerical and experimental results involves the slope of the heave emerging on the right of the caisson. The numerical analysis gives a significantly steeper heave because it does not allow for the soil on the heave to slide and flow around the foundation, as it happens during the experiment. The predicted caisson response is presented in Figure 5.58 in view of the rotational and translational displacements with respect to fault throw and the results are compared to the test.

5.5.2.3 Rupture – Caisson Interaction: \( s/B = -0.04 \) (Reverse Fault)

In this case, similarly to the experimental results and as shown in Figure 5.59, the rupture zone reaches the bottom right corner and then is partitioned in two branches, one going to the right and one going to the left of the foundation. The base displacement required for the rupture to reach the surface according to the prediction of the numerical analysis is quite close to the value measured through the physical model testing. In Figure 5.60, the finite element analysis calculation of the surface deformation is compared with the measured data. The comparison is quite good although the analysis tends to overestimate the rotation of the caisson for \( h > 1.5 \) m (Fig. 5.61).
5.5.2.2.4  Rupture – Caisson Interaction: $s/B = 0.66$ (Reverse Fault)

In accordance with the experimental results, the numerical analysis indicates the development of two failure mechanisms (Fig. 5.62), one on each side of the caisson (fault diverted to the left of the caisson and sliding plane along its right side). The agreement with the experimental results is quite satisfactory and the numerical method seems to capture the FRSFI mechanisms that take place. In the numerical analysis, however, the imposed deformation $h$ for the rupture to reach surface is larger than 1.5m.

The effectiveness of the numerical method in capturing the FRSFI mechanisms is graphically depicted in Figure 5.63, which presents (out of scale) the deformed shape of the model surface for different values of fault dislocation compared with the corresponding data from the measurements during the experiment. Despite the discrepancy in the prediction of the deformed surface on the left of the caisson, the movement of the caisson and the heave formation on the right of it are sufficiently reproduced. This is also reflected in Figure 5.64 where the measured and the calculated foundation rotation and displacements are illustrated.

5.5.3  Synopsis – Conclusions

A 3D numerical methodology is applied and validated through comparison with 1g experiment data. Although the numerical method is not capable of capturing the strain localization along the rupture paths in full detail, the analysis reproduces quite sufficiently the FRSFI mechanisms involved in the problem, as well as the consequent displacements and rotations of the caisson. Thus, the numerical method can be characterized as an effective tool for the study of such a complicated problem.

In the following, the numerical methodology described above is utilized for the development of a simplified methodology for the estimation of the fault-induced stressing on the whole structure (foundation and superstructure).
Figure 5.43. Snapshots of the deformed model for characteristic values of the fault movement. Comparison of experiment and analysis [normal fault – free field] (1st row: snapshots from the experiment; 2nd row: deformed shape and plastic strain contours from the analysis).

Figure 5.44. Model surface deformed shape for characteristic values of the fault movement. Comparison of experiment and analysis [normal fault – free field].
Figure 5.45. Snapshots of the deformed model for characteristic values of the fault movement. Comparison of experiment and analysis [normal fault – caisson with s/B = 0.16] (1st row: snapshots from the experiment; 2nd row: displacement vector field from the experiment; 3rd row: deformed shape and plastic strain contours from the analysis).
Figure 5.46. Model surface deformed shape for characteristic values of the fault movement. Comparison of experiment and analysis [normal fault – caisson with s/B = 0.16].

Figure 5.47. Evolution of the caisson displacements and rotation as a function of fault movement $h$. Comparison of experiment and analysis [normal fault – caisson with s/B = 0.16].
Figure 5.48. Snapshots of the deformed model for characteristic values of the fault movement. Comparison of experiment and analysis [normal fault – caisson with s/B = 0.38] (1st row: snapshots from the experiment; 2nd row: displacement vector field from the experiment; 3rd row: deformed shape and plastic strain contours from the analysis).
Figure 5.49. Model surface deformed shape for characteristic values of the fault movement. Comparison of experiment and analysis [normal fault – caisson with s/B = 0.16].

Figure 5.50. Evolution of the caisson displacements and rotation as a function of fault movement $h$. Comparison of experiment and analysis [normal fault – caisson with s/B = 0.38].
Figure 5.51. Snapshots of the deformed model for characteristic values of the fault movement. Comparison of experiment and analysis [normal fault – caisson with s/B = 0.80] (1st row: snapshots from the experiment; 2nd row: displacement vector field from the experiment; 3rd row: deformed shape and plastic strain contours from the analysis).
Figure 5.52. Model surface deformed shape for characteristic values of the fault movement. Comparison of experiment and analysis [normal fault – caisson with s/B = 0.16].

Figure 5.53. Evolution of the caisson displacements and rotation as a function of fault movement \( h \). Comparison of experiment and analysis [normal fault – caisson with s/B = 0.80].
Figure 5.54. Snapshots of the deformed model for characteristic values of the fault movement. Comparison of experiment and analysis [normal fault – free field] (1st row: snapshots from the experiment; 2nd row: deformed shape and plastic strain contours from the analysis).

Figure 5.55. Model surface deformed shape for characteristic values of the fault movement. Comparison of experiment and analysis [reverse fault – free field].
Figure 5.56. Snapshots of the deformed model for characteristic values of the fault movement. Comparison of experiment and analysis [reverse fault – caisson with s/B = -0.96] (1st row: snapshots from the experiment; 2nd row: displacement vector field from the experiment; 3rd row: deformed shape and plastic strain contours from the analysis).
Figure 5.57. Model surface deformed shape for characteristic values of the fault movement. Comparison of experiment and analysis [reverse fault – caisson with s/B = -0.96].

Figure 5.58. Evolution of the caisson displacements and rotation as a function of fault movement h. Comparison of experiment and analysis [reverse fault – caisson with s/B = -0.96].
Figure 5.59. Snapshots of the deformed model for characteristic values of the fault movement. Comparison of experiment and analysis [reverse fault – caisson with s/B = -0.04] (1st row: snapshots from the experiment; 2nd row: displacement vector field from the experiment; 3rd row: deformed shape and plastic strain contours from the analysis).
Figure 5.60. Model surface deformed shape for characteristic values of the fault movement. Comparison of experiment and analysis [reverse fault – caisson with s/B = -0.04].

Figure 5.61. Evolution of the caisson displacements and rotation as a function of fault movement $h$. Comparison of experiment and analysis [reverse fault – caisson with s/B = -0.04].
Figure 5.62. Snapshots of the deformed model for characteristic values of the fault movement. Comparison of experiment and analysis [reverse fault – caisson with s/B = 0.66] (1st row: snapshots from the experiment; 2nd row: displacement vector field from the experiment; 3rd row: deformed shape and plastic strain contours from the analysis).
Figure 5.63. Model surface deformed shape for characteristic values of the fault movement. Comparison of experiment and analysis [reverse fault – caisson with s/B = 0.66].

Figure 5.64. Evolution of the caisson displacements and rotation as a function of fault movement $h$. Comparison of experiment and analysis [reverse fault – caisson with s/B = 0.66].
5.6 SIMPLIFIED METHOD FOR ESTIMATION OF THE FAULTING–INDUCED STRESSING ON STRUCTURES

In this section a methodology for the estimation of the structure stressing due to faulting is proposed based on the findings from the experimental and the analytical work described above. The methodology proposed here has been developed for bridges (as they are the most typical example of structures founded on caissons); nevertheless with minor modifications it can be used for various structures as well.

According to the proposed methodology, the analysis of the bridge–foundation system subjected to faulting–induced deformation is conducted in two steps, in which the interaction between rupture, soil, foundation, and superstructure is rationally taken into account. Specifically:

In Step 1 (local level), we analyze the response of a single foundation member (i.e. caisson, pile, pile-group, footing, etc.) subjected to fault rupture deformation. A detailed model (like the finite element one described above) is employed for the aforementioned fault rupture soil–foundation–structure interaction (FR-SFSI), with the superstructure modeled in a simplified manner: it is replaced by equivalent vertical, lateral and rotational springs, \( K_z \), \( K_x \) and \( K_\theta \), respectively, representing the stiffness of the superstructure. The output of this step is dual: (i) it provides information regarding the distress of the foundation system (e.g. the internal forces in piles, in case of a piled foundation); (ii) it provides the necessary input for the second step: the horizontal and vertical displacements \( \Delta x \) and \( \Delta y \) and the rotation \( \theta \) at the base of the superstructure.

In Step 2 (global level), the detailed model of the superstructure is subjected to the computed \( \Delta x \), \( \Delta y \), and \( \theta \) from Step 1.

The methodology is summarized and graphically presented in Figure 5.65.
Figure 5.65. Proposed two-step methodology for the estimation of structural stressing due to faulting. In Step 1, we analyze the response of a single foundation subjected to fault rupture deformation. A detailed model is employed to model fault rupture soil–foundation–structure interaction (FRSFSI), with the superstructure modeled in a simplified manner (replaced by...
springs). In Step 2, the detailed model of the superstructure is subjected to the computed displacements and rotations of Step 1.

5.7 SYNOPSIS – CONCLUSIONS

As part of Phase VI of the JRA 3.4, the Laboratory of Soil Mechanics (LSM) of NTUA has conducted a series of 1g experiments in the Fault Rupture Box (FRB), to investigate fault rupture propagation through soil and its interaction with embedded caisson foundations. The following parameters have been investigated: (a) the style of dip-slip faulting (normal and reverse), and (b) the position of the foundation relative to the fault rupture. The "bedrock" was subjected to movement due to fault rupture of vertical offset $h$ at a dip angle of 45°. To investigate the effect of the location of the structure relative to the fault rupture, the foundation was parametrically positioned at distance $s$ from the outcropping location of the unperturbed (i.e. free field) fault rupture. The displacements of the foundation, $\Delta x$, $\Delta z$, and the rotation $\theta$, as well as the deformation of the soil mass were recorded during the experiments through image analysis and a laser scanning of the soil surface. In the first case, digital high-resolution cameras are utilized to capture photographs during the incrementally imposed fault rupture displacement $h$, which are then processed through image analysis. In the latter case, a novel technique was developed. After each displacement increment, the model surface was scanned with 8 laser displacement transducers, which travel along the specimen at a constant speed, producing a digital relief of the deformed surface.

Moreover, an attempt was made to numerically simulate the conducted experiments. Analyses using finite element models were carried out and the results are quite encouraging.

Finally, a methodology has been proposed for the estimation of the structural stressing due to faulting.

The concluding remarks drawn by the present study are summarized below.
The caisson foundations strongly interact with dip-slip fault rupture, both normal and reverse, and alter the rupture propagation path compared to the free field case. The caisson acts as a rigid body and imposes a kinematic constrain to the fault rupture.

The critical parameter that controls the response of the system is the foundation position relatively to the free field rupture. Different FRSFI mechanisms were observed for different positions of the caisson and have been described in detail. Due to the strong interaction of the caisson with the fault rupture, the performance of the former varied from minor settlement to significant displacements and rotations.

The numerical method used was proven to be capable of simulating such a difficult problem with sufficient accuracy.

Based on the findings of the experiments and the efficiency of the numerical analysis, a two-step methodology has been proposed for the calculation of the fault-induced stressing on structures. According to this methodology, in the first step, the response of the foundation itself is calculated through numerical analyses like these presented in this study, and then, in the second step, the performance of the superstructure is estimated through analysis of the detailed model of the structure imposed to the displacements and rotations calculated during the first step.
6 SUMMARY

6.1 SOIL CONTAINERS FOR STRONG MOTION TESTING

An evaluation of the performance of existing shear stacks at large deformations is presented. It seems that to model the response of soils at both small and large deformations it is necessary to minimize the stiffness and inertia of the soil container. Based on this observation, a conceptual design of a new shear stack intended to capitalize on this realization is presented.

The main problem with current designs for lamellar boxes is the low manufacturing tolerance that is required for the stack to “shear” without any sticking and the high cost of the bearings. Therefore as an alternative to the standard rigid ring model for a shear stack, a conceptual design of a shear stack is considered that has end walls consisting of a set of horizontal parallel beams each supported by two passive hydraulic pistons. With an appropriate arrangement of hydraulic piping and valves, in principle it is possible to lock the stack rigid, to have a fully flexible stack (but with the beams remaining parallel and tracking each other), to have a flexible stack but with increased damping, to have adjustable stiffness in the ends of the stack and finally to have an actuated stack. This system can also be modified to provide the parallel motion of the end beams but with more flexibility over the stiffness of each layer at each end of the stack. However, although the concept of a hydraulic shear stack shows some promise, and certainly opens up some experimental options that are not available in current designs, the hydraulics would need very careful design in order to minimise the energy losses in the system and make such a system practicable.
6.2 APPLICATION OF FHT TECHNIQUE AT NTUA

A novel framework for effective RTDS of the horizontal SSI problem, based on adaptive signal processing and parameter estimation techniques has been developed at LEE – NTUA, the main features of which are: (i) an adaptive controller is designed and identified, either off – line, or on – line. Although this procedure may increase the delay, it is unavoidable when the transfer system has its own delay and/or it is minimum phase; (ii) a predictor is placed prior to the numerical substructure to compensate the effects of the delay. This predictor handles the part of the relative acceleration of the specimen that cannot be a priori known; the process is designed to work in acceleration mode: not only acceleration commands are given to the transfer system, but acceleration feedback is also utilized, dropping the need for load cells.

The research is ongoing, as several problems that were addressed are currently being investigated. Among these are: the need for automatic gain controllers that must be placed in the adaptation process, so as to assure that the cascade of the estimated FIR filters is indeed a delayed version of the Kronecker’s delta function; the need for integration of the current experimental facility and effective signal processing units; the need for a systematic identification procedure for the specimen, so as to include time – varying and nonlinear structures.

In any case, the results are promising and show that the proposed scheme may lead to a robust RTDS method.

6.3 APPLICATION OF FHT TECHNIQUE AT UNIVBRIS

An integrated model of a shaking table test system has been developed by linearising various mathematical models of the constituent parts (namely the electro-hydraulic servovalves, the hydraulic actuators and the proprietary controller). A linear system analysis using the integrated model reveals that while either a third or fourth order system can be used to adequately describe the shaking table system, the fourth order system gives a better representation of the shaking table system across a wider bandwidth (i.e. 0 to 20Hz). The integrated shaking table model has
been used to develop a model of an entire shaking-table substructuring test system which demonstrates how phase lag in the transfer system can have a detrimental influence on stability. This well-known finding has prompted the development and widespread use of delay compensation outer loop control algorithms which have been found to enhance both accuracy and stability of substructuring tests that use a single actuator as the transfer system. Finally, a number of substructuring tests were conducted to assess the efficacy of such a delay compensation algorithm on substructuring tests using the shaking table at the University of Bristol as the transfer system. The test results show that in some cases delay compensation can improve the stability and accuracy by reducing the phase lag of the shaking table. However, in other cases, delay compensation is shown to be unsuitable for shaking table substructuring by, contrary to conventional wisdom, diminishing the stability.

To understand these unintuitive results, a method that can predict the stability boundaries of a structured system that takes into account the dynamics of a shaking table has been developed based on the roots locus technique. The validity of the method has been verified experimentally. The method has been applied to assess the effect on stability of parametric variations within a two DOF shaking table substructuring system. The analytical results reveal that the influence on stability of varying the properties of the physical substructure is contrary to that when varying the properties of the numerical substructure and that the properties of physical substructure have a dominant influence on the stability.

A rational transfer function for delay compensation is obtained by Padé approximation that has been used to analyse the performance of a delay-compensated shaking table substructuring system. The stability is investigated using the roots locus technique and the accuracy is investigated by analytically and experimentally. Delay compensation is demonstrated to be functional for shaking table substructuring only for low frequency and damping systems subjected to low frequency excitation; accuracy and stability diminish as the damping and frequency increase.

A new control methodology has been developed to improve on the deficiencies of delay compensation and to enable substructuring of the parameter set ordained by SERIES (i.e. that
includes testing a physical substructure which resonates at 5Hz. Full State Control via Simulation (FSCS) for substructuring is a conjunction of inverse dynamic control and full state feedback control. FSCS can compensate for both the phase lag and magnitude errors associated with complex transfer systems such as shaking tables. The stability of FSCS is investigated using the roots locus technique. The analytical results show that the FSCS controller brings a significant stability enhancement. The accuracy of FSCS has been demonstrated experimentally. FSCS can be used successfully for shaking table substructuring when the substructures have high frequency and low damping and when the excitation frequency attains high magnitudes. FSCS has been applied to shaking-table-substructuring SSI studies where it has been successfully implemented with linear lumped mass and nonlinear macroelement numerical substructures.

6.4 INVESTIGATION OF FAULT RUPTURE PROPAGATION

A series of 1g experiments in the Fault Rupture Box (FRB) have been conducted, in order to investigate fault rupture propagation through soil and its interaction with embedded caisson foundations. The following parameters have been investigated: (a) the style of dip-slip faulting (normal and reverse), and (b) the position of the foundation relative to the fault rupture. The "bedrock" was subjected to movement due to fault rupture of vertical offset \( h \) at a dip angle of 45° while the foundation was parametrically positioned at distance \( s \) from the outcropping location of the unperturbed fault rupture.

The displacements of the foundation, \( \Delta x, \Delta z \), and the rotation \( \theta \) were recorded during the experiments through image analysis and a laser scanning of the soil surface. For the measurement of the deformation of the soil mass, a novel technique was developed. After each displacement increment, the model surface was scanned with 8 laser displacement transducers, which travel along the specimen at a constant speed, producing a digital relief of the deformed surface.

Moreover, an attempt was made to numerically simulate the conducted experiments using finite element models with encouraging results. Finally, a methodology has been proposed for the estimation of the structural stressing due to faulting.
This study showed that the caisson foundations strongly interact with dip-slip fault rupture, both normal and reverse, and alter the rupture propagation path compared to the free field case. The critical parameter that controls the response of the system is the foundation position relatively to the free field rupture. Due to the strong interaction of the caisson with the fault rupture, the performance of the former varied from minor settlement to significant displacements and rotations.

Based on the findings of the experiments and the efficiency of the numerical analysis, a two-step methodology has been proposed for the calculation of the fault-induced stressing on structures. In the first step, the response of the foundation itself is calculated through numerical analyses; in the second step, the performance of the superstructure is estimated through analysis of the detailed model of the structure imposed to the displacements and rotations calculated during the first step.
Shaking table test techniques and fault rupture box testing for SSI
7 REFERENCES


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